Why the Political World is Flat: An Endogenous "Left" and "Right" in Multidimensional Elections

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Abstract

This paper analyzes candidate positioning in multidimensional, common-interest elections. Candidates can polarize in various directions in equilibrium, giving structure to the common concern that issues might be bundled inefficiently. However, the only stable equilibrium efficiently bundles issues that are logically related. Consistently bundling issues in this way can explain why multidimensional policy decisions invariably reduce to the same single dimension.

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1 Introduction

The world of public policy is complex and multifaceted. Elected officials must decide tax policy, foreign policy, health policy, education policy, immigration policy, social policy, and numerous more narrow issues that are themselves complex and multifaceted. Indeed, every sentence of legislation could be viewed as a separate dimension, along which policy could be adjusted. Voters must consider all of these as well, along with candidate characteristics such as honesty and management skill. The number of dimensions required to properly model such an environment is enormous. In contrast, existing political economic models are almost exclusively one dimensional.

Efforts to understand multidimensional politics have repeatedly been stymied by one of three challenges. Some models exhibit no equilibrium, at least in pure strategies.[1] Some have numerous equilibria.[2] Others make the unique but counter-factual equilibrium prediction that candidates will adopt identical policies.[3] One way to interpret this literature is as a prediction of "chaos": challengers should tend to unseat incumbents, contests between symmetric challengers should be unpredictable, and successive majority votes may cycle indefinitely through the same policies, or lead to *any* eventual policy outcome (McKelvey, 1979). Tullock (1981) points out, however, that such instability is not evident empirically.[4]

[1]These include Plott's (1967) straightforward extension of the classic one-dimensional model of Hotelling (1929) and Downs (1957). See also Duggan and Fey (2005). Mixed-strategy equilibria exist more generally (Duggan and Jackson, 2005), but have unclear empirical relevance in the context of political campaigns (Austen-Smith and Banks, 2005).

[2]For example, see the "citizen-candidate" models of Osborne and Slivinski (1996) and Besley and Coate (1997), where "basically *any* pair of candidates who split the voters evenly can be an equilibrium."

[3]Examples include probabilistic voting models such as Hinich (1978). More recently, see Xefteris (2017). That candidates remain quite polarized, empirically, is documented by Bafumi and Herron (2010) and Shor (2011), among others.

[4]Mueller (2003, ch. 11) reports, for example, that incumbent U.S. governors have historically won reelection by an average margin of 23%.

The more common response to this literature is simply to continue using one dimension—in essence, treating the world as if it were flat (in fact, one dimensional) when it is known not to be. Actually, this approach has some merit: empirically, preferences across issues are surprisingly correlated, and so are effectively summarized by one dimension. In the words of Converse (1964, p. 207), "...if a person is opposed to the expansion of social security, he is probably a conservative and is probably opposed as well to any nationalization of private industries, federal aid to education, sharply progressive income taxation, and so forth." Poole and Rosenthal (1985, 1997, 2001) formalize this statistically, showing that one dimensional correctly predicts almost 90% of the roll call votes cast in the U.S. House and Senate. They cite similar findings for the European Parliament, the U.N. General Assembly, and several European national parliaments, and Grofman and Brazill (2002) and Shor and McCarty (2011) find the same for the U.S. supreme court and state legislatures. Converse (1964) views voters as less ideologically consistent than politicians, but Shor (2014) finds that a single dimension explains 80% of voter opinions, as well.[5]

By modeling voter differences as immutable taste parameters, the literature above embraces a *private interest* paradigm. In McMurray (2017a) I argue that many aspects of voter behavior fit more neatly within a *common interest* paradigm, meaning that voters favor whatever is truly best for the group, but disagree which welfare consequences a policy will have. The fundamental hope of democracy is then to distinguish truth from the myriad of opinions, as in the classic "jury" theorem of Condorcet (1785).[6] In one dimension, I show in McMurray (2018) that common

[5]Preferences also correlate on local and national issues (Tausanovitch and Warshaw, 2014).

[6]Instances of selfless voting abound, like the wealthy who favor redistribution to the poor. As that paper explains, large elections can amplify altruism, so that almost purely selfish voters base their votes mostly on (their perceptions of) the public interest. The fragility of information explains why opinions evolve over time and why voters try to persuade others to their points of view, and expect to win. A spatial common interest model also explains why information and ideology correlate empirically, contrary to the predictions of private interest models.

interest voting leads candidates to polarize substantially, even when they are highly motivated to win, because a candidate who believes that truth is on her side expects voter support even when she is more extreme than her opponent.[7] This paper extends that work to multiple dimensions.

An immediate benefit of treating voters' policy differences as a matter of opinion rather than taste is a plausible rationale for why voter attitudes should be correlated across issues. Quite simply, the same logical considerations that favor one policy also favor another. For the purposes of ending an economic recession, for example, it might turn out that fiscal stimulus is effective while monetary stimulus is not, or vice versa, but ex ante it is more likely either that both forms of stimulus are beneficial (because the economy functions more or less as Keynesian models predict) or that both are wasteful (as in more classical models), so support for one form of stimulus is likely to be correlated with support for the other.[8]

The equilibrium analysis below makes clear that polarizing forces operate in higher dimensions, as well. With multiple dimensions, however, candidates could polarize in infinitely many directions, bundling any combination of issues. In a perfectly symmetric specification of the model, equilibrium could sustain any of these, so indeterminacy remains a severe problem. However, correlation in voter opinions breaks symmetry, so the number of equilibria falls precipitously. In two dimensions, for example, only two equilibria remain, oriented along the major and minor diagonals of the policy space. The latter is inefficient, thus formalizing the prevalent concern that issues are bundled sub-optimally. That equilibrium is unstable, however, so the major equilibrium emerges as the unique prediction of the model.

Empirically, issue positions that are bundled together as "liberal" or "conserva-

[7] In this paper, masculine and feminine pronouns refer to voters and candidates, respectively.

[8] Similarly, belief or disbelief in market efficiency or in the competence and integrity of government regulators may jointly determine a voter's support (or lack thereof) for a host of regulations. Beliefs about the relative importance of luck and effort in determining individual fortunes could shape a voter's support for a variety of redistribution.

tive" in a particular election, place, and time, tend to be bundled together elsewhere, as well. For example, a single dimension categorizes nearly 90% of the policy positions of eighty political parties in seventeen countries, over twenty-five years (McDonald, Mendes, and Kim, 2007). Similar ideological structure arises even in authoritarian China, suggesting that attitude correlation is not driven by political institutions (Pan and Xu, 2017).[9] Such stability is not easily explained by existing explanations of unidimensionality, but is consistent with the information model below, as logic that links two issues in one setting should link the same issues in other settings, as well.

In many cases, logical relationships between issues may seem too weak to drive the consistent bundling of issues across elections. Below, however, *any* non-zero correlation between truth variables is sufficient to orient the equilibrium in the direction of correlation, leading candidates to behave just as they would if the correlation were perfect. Consistent with the data, the model also predicts that candidates should be more ideologically consistent than voters.

Information models are challenging to analyze, because each voter and candidate must forecast everyone else's private information. Multidimensionality notoriously complicates things as well. At a desirable level of generality, therefore, the multidimensional information model below does not seem fully tractable. To make unambiguous predictions, the baseline model imposes a large amount of symmetry and other restrictions on various model primitives. Section 5 then conducts a partial analysis of a more general model, making clear that these restrictions are unnecessary for the paper's central results, and serve only to clarify the underlying theoretical mechanisms. To keep things as clear as possible, the model of Section 3 also assumes voter preferences to be identical, but Section 7 explains that a mix of private and common interests would generate similar results.

[9] Political factions (e.g. Tories and Whigs in England, Federalists and Anti-Federalists in the U.S.) also predate formal political parties (https://en.wikipedia.org/wiki/Political_party). Scientific and other non-political groups divide similarly into competing "schools of thought".

2 Literature

Private interest election models typically attribute ideology to exogenous tastes, or to differences in wealth, which determines the demand for redistribution (Romer, 1975; Meltzer and Richard, 1981) and public goods (Bergstrom and Goodman, 1973). With multiple public goods or redistributive policies, this might implicitly justify unidimensionality, as well. Empirically, however, the wealthy favor redistribution almost as frequently as the poor, as I discuss in McMurray (2017a), and public goods such as defense and environmental protection draw support from opposite groups of voters. Private interest models also face the challenges outlined in Section 1.

In private interest elections with non-binding policy platforms, Schnakenberg (2016) shows that cheap talk only credibly reveals which side of a hyperplane a candidate's preferred policy is on. As in the most symmetric model below, however, there are infinitely many such equilibria, giving no reason to expect the same policy bundling across elections. In the communication literature, Spector (2000) considers two groups with different prior beliefs about a commonly valued, multidimensional state variable. As these communicate over time, their beliefs converge in every direction except the one of prior disagreement, where communication lacks credibility. This direction is exogenous, however, again giving no reason for the same issues to be bundled together consistently.[10] DeMarzo, Vayanos, and Zwiebel (2003) assume that individuals repeatedly circulate their initial information through a social network, but fail to rationally discount repeated information (see also Louis, Troumpounis, and Tsakas, 2018). The direction of slowest consensus can be interpreted as the left and right of politics, but again, settings with different initial beliefs or network structures should then bundle issues differently.[11]

[10] This also relies on the restriction to two homogeneous groups: with additional groups or heterogeneous groups, full information could be inferred from the cross-section of others' messages, as in Battaglini (2002).

[11] Duggan and Martinelli (2011) and Egorov (2014) show how the orientation of political conflict can be influenced by a monolithic media or by candidate messaging, respectively, but these take

Though neither address candidate positioning or unidimensionality, two papers study information aggregation in multidimensional common-interest settings. Feddersen and Pesendorfer (1997) show that conflicts of interest impede information aggregation when there are more dimensions than one. Barelli, Bhattacharya, and Siga (2015) identify conditions on the information structure that, in the absence of conflict, are necessary and sufficient for efficient information aggregation in arbitrary dimensions.

3 The Model

A society consists of N voters, where N is drawn from a Poisson distribution with mean $n \in \mathbb{N}$. Together, these voters must choose a pair $x = (x_1, x_2)$ of policies from the unit disk, X.[12] It is often convenient to instead represent policies using polar coordinates (r_x, θ_x), or as a column vector $x = \binom{x_1}{x_2}$, where multiplying by $R_\theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ then produces a *rotation* $R_\theta x$ which has the same magnitude as x, but polar angle $\theta_x + \theta$, and multiplying by $M_\theta = \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$ produces the *mirror image* through angle θ, which has the same magnitude as x but polar angle $2\theta - \theta_x$, so that x and $M_\theta x$ are equidistant from a vector with angle θ.

Within the set of feasible policy bundles, one is ultimately socially optimal. Denote this as $z = (z_1, z_2)$ or $z = (r_z, \theta_z)$, and assume that every voter prefers policy pairs as close as possible to z. The set of policies that might be optimal is $Z = X$.[13]

unidimensionality as an exogenous constraint on communication.

[12] $(0, 0)$ might represent a pair of status quo policies, for example, and the electorate can depart from this in any direction up to some maximal distance, normalized to one. A disk is more convenient than, say, a Cartesian product $X = [-1, 1]^2$, because its symmetry greatly simplifies the analysis. Section 5 discusses more general X, as well as $K > 2$ dimensions.

[13] Section 5 allows that certain feasible policies are known *not* to be optimal, $Z \subset X$.

Voter utility

$$u(x,z) = -\|x-z\|^2 = -(x-z)\cdot(x-z) \tag{1}$$

decreases quadratically in the Euclidean distance $\|x-z\|$ between x and z. Conditional on information Ω (and dropping terms that do not depend on the policy outcome), expected utility

$$E[u(x,z)|\Omega] = -\|x - E(z|\Omega)\|^2 \tag{2}$$

then decreases quadratically in the distance between the policy vector implemented and the updated expectation $E(z|\Omega)$ of the optimum.[14]

As Section 1 explains, z_1 and z_2 should be correlated, because of logical connections across issues. To allow this possibility, let the prior density $f(z;\rho)$ depend on a parameter ρ related to the correlation between z_1 and z_2. In fact, to make the analysis unambiguous, let f satisfy *correlative monotonicity* (Condition 1). For positive ρ, this means that f increases in the direction of the *major diagonal* (i.e. the line defined by $z_1 = z_2$) and decreases in the direction of the *minor diagonal* (i.e. defined by $z_1 = -z_2$), and that this pattern becomes more pronounced as ρ increases. Additionally, assume that f exhibits *dimensional symmetry* (Condition 2), meaning that f is symmetric around the origin, and that reorienting one dimension and reversing the sign of ρ are equivalent. Figure 1 illustrates an example

$$f(z;\rho) = \frac{1}{\pi}\left(1+\rho\frac{z_1 z_2}{\|z\|}\right) = \frac{1}{\pi}\left[1+\rho r_z \cos(\theta_z)\sin(\theta_z)\right] \tag{3}$$

that satisfies Conditions 1 and 2, where $\rho \in [-2,2]$ and the correlation coefficient between z_1 and z_2 equals $\frac{\rho}{5}$.

Condition 1 (Correlative monotonicity) *$f(z_1, z_2; \rho)$ is differentiable in z_1, z_2, and ρ. Moreover, $\frac{\partial f(z)}{\partial z_1}$ and $\frac{\partial^2 f(z)}{\partial z_1 \partial \rho}$ have the same signs as z_2 and ρz_2, respectively,*

[14] The quadratic specification here is convenient but not essential. The important feature of (2) is that shifts in the distribution of z shift the desired policy in the same direction. With linear utility, for example, a voter would favor the median realizations of z_1 and z_2 (conditional on Ω) instead of the mean, with similar implications for behavior.

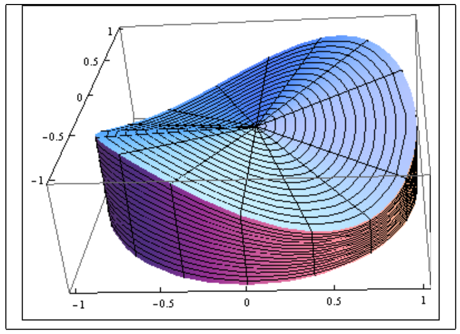

Figure 1: An example density that satisfies Conditions 1 and 2.

and, symmetrically, $\frac{\partial f(z)}{\partial z_2}$ *and* $\frac{\partial^2 f(z)}{\partial z_2 \partial \rho}$ *have the same signs as* ρz_1 *and* z_1 *(implying that* $\frac{\partial f(z)}{\partial \theta_z}$ *has the same sign as* $\rho \cos(2\theta_z)$*). Also,* $\frac{\partial^2 f(z)}{\partial z_1 \partial z_2}$ *has the same sign as* ρ*,* $\frac{\partial f(z)}{\partial \rho}$ *has the same sign as* $z_1 z_2$ *(and* $\sin(2\theta_z)$*), and* $\frac{\partial^2 f(z)}{\partial \theta_z \partial \rho}$ *has the same sign as* $|z_1| - |z_2|$ *(and* $\cos(2\theta_z)$*).*

Condition 2 (Dimensional symmetry) $f(z_1, z_2) = f(z_2, z_1) = f(-z_1, -z_2)$ *and* $f(-z_1, z_2) = f(z_1, z_2; -\rho)$*. Equivalently,* $f(z) = f\left(M_{\frac{\pi}{4}} z\right) = f(R_\pi z)$ *and* $f\left(M_{\frac{\pi}{2}} z\right) = f(M_0 z) = f\left(R_{\frac{\pi}{2}} z\right) = f(z; -\rho)$*.*

Correlative monotonicity implies that, when $\rho = 0$, f is uniform and thus satisfies *radial symmetry* (Condition 3), meaning that the optimal policy pair is equally likely to lie in any direction from the origin.

Condition 3 (Radial symmetry) $f(R_\theta z) = f(M_\theta z) = f(z)$ *for any* $\theta \in \mathbb{R}$ *and for any* $z \in Z$*.*

Opinions regarding the location of the optimal policy pair are determined by pairs $s_i = (s_{i1}, s_{i2})$ of informative private signals, drawn independently (conditional on z) from the set $S = Z$ of possibly optimal policy pairs. Intuitively, s_{i1} should be informative of z_1 and s_{i2} should be informative of z_2. Both to accomplish this and so that posterior beliefs can be tractably characterized, assume that the conditional density $g(s|z)$ of private signals satisfies *linear informativeness* (Condition 4), meaning that $g(s|z)$ is linear, sloping upward in the direction of z.

Condition 4 (Linear informativeness) *$g(s|z) = g_0 + g_1(s \cdot z)$ for some $g_0, g_1 > 0$.*

Implicitly, linear informativeness implies two forms of symmetry on the distribution of signals. *Rotational symmetry* (Condition 5) means that rotating z rotates the entire distribution of signals by the same amount. *Error symmetry* (Condition 6) means that a signal s is equally likely to deviate from z in a clockwise or counter-clockwise direction. Beyond this symmetry, the technical importance of linearity is that it makes posterior beliefs tractable even after voters update to account for the event of a pivotal vote, and ensures that these posterior beliefs depend monotonically on voters' signals, so that best-response voting is similarly monotonic.[15] Symmetry and linearity facilitate a more complete characterization of equilibrium behavior, but as Section 5 explains, do not seem important for the fundamental logic of any of the results below.

Condition 5 (Rotational symmetry) *$g(R_\theta s|R_\theta z) = g(s|z)$ for any θ.*

Condition 6 (Error symmetry) *$g(M_{\theta_z} s|z) = g(s|z)$ and $g(s|M_{\theta_s} z) = g(s|z)$.*

Condition 2 implies that $E(z) = 0$, so the linear informativeness of g implies that s_i is uniform: $g(s) = E[g(s|z)] = g_0 + g_1 s \cdot E(z) = g_0$. The posterior expectation of z is then linear in s.[16]

$$E(z_1|s) = \int_Z z_1 \frac{g_0 + g_1(s_1 z_1 + s_2 z_2)}{g_0} f(z)\, dz$$

[15] An alternative assumption that would deliver similar results is that signals are informative and voters vote *sincerely* on the basis of s alone, foregoing any additional information revealed in equilibrium by other voters' behavior. The advantage of the present formulation is simply in making clear that monotonic voting can be rational, at least for a stylized information distribution, even when voters take this additional information into account. Rational voting is important for the sake of application, and also delivers efficient information aggregation, which sincere voting generally does not (McMurray 2018).

[16] For the densities (3) and (**??**), for example, $E(z|s) = \frac{1}{4}\binom{s_1+\rho s_2}{\rho s_1 + s_2}$.

$$
\begin{aligned}
&= \frac{g_1}{g_0} V(z_1)(s_1 + \rho s_2) &(4)\\
E(z_2|s) &= \frac{g_1}{g_0} V(z_2)(\rho s_1 + s_2) &(5)
\end{aligned}
$$

Naturally, $E(z_1|s)$ increases in s_1 and $E(z_2|s)$ increases in s_2; if ρ is positive then $E(z_1|s)$ also increases in s_2 and $E(z_2|s)$ also increases in s_1.[17] The distribution of signals is continuous, so despite their common objective, voters develop a myriad of different opinions about which policy combination is optimal.

Candidates A and B choose platforms $x_A = (x_{A1}, x_{A2})$ and $x_B = (x_{B1}, x_{B2})$ in X, and voters each vote for one of these. The candidate $w \in \{A, B\}$ who receives the most votes (breaking ties, if necessary, by a coin toss) wins the election (event $\Pr(A)$ or $\Pr(B)$) and implements her platform policies. In choosing policies, candidates are assumed to be *truth motivated*, meaning that, like voters, they maximize (2), desiring the final policy outcome to be as close as possible to whatever is truly optimal.[18] This is parsimonious in that candidates are fundamentally no different from other citizens.[19]

Given their career choice in policy making and privileged access to policy advice, it is reasonable that candidates should observe higher quality signals of z than the typical voter. To behave optimally, however, each voter and candidate must infer

[17] The assumption that signals on one issue are informative of another issue is consistent with recent evidence from Brunner, Ross, and Washington (2011) that economic conditions have a causal impact on both economic and non-economic vote choices.

[18] In McMurray (2017b) I show that truth motivated candidates can adjust their policy positions after an election, utilizing additional information revealed by the margin of victory. In the present model, voters have no reason to prohibit this, since candidates are known to share their preferences. A culture of enforcing platform commitments may be warranted, however, if there is positive probability of candidates holding deviant preferences (as I explore in McMurray 2018). Binding commitments also make the analysis more directly comparable with existing literature, and avoid the complexities of forecasting candidates' ex post behavior, limiting to the number of policy outcomes to one per candidate.

[19] Section 5 comments on the behavior of candidates who also value winning. Section 7 discusses deviations from purely common policy interests.

whatever possible from the other actors in society. When all receive signals, this leads to higher order beliefs that seem hopelessly intractable. To avoid these complexities, candidate signals are not modeled, and anything they infer from voters constitutes their *only* source of information. As in McMurray (2018), however, such inference turns out to be quite strong. In fact, Section 5 explains that candidates' signals—no matter how precise—would be superfluous when the number of voter signals is already large.

In real-world elections, of course, voters observe candidate platforms before voting. To induce symmetry that makes the analysis more completely tractable, however, voters and candidates in this model instead move simultaneously. This actually doesn't matter for voter behavior, which in equilibrium best-responds to candidate platforms either way. In large elections, it doesn't matter for candidates, either, as Section 5 explains below. With simultaneous voting, the appropriate solution concept is Bayesian Nash equilibria (BNE). In addition, I focus on equilibria in which voters respond identically to identical signals and platforms, and candidates platforms are symmetric.[20] Such equilibria can be characterized by platforms x_A and x_B together with a voting strategy $v : S \to \{A, B\}$ (from the set V) that specifies a vote choice for every signal vector $s \in S$ but does not depend explicitly on x_A and x_B.

4 Equilibrium Analysis

The analysis of voter responses to $x_A, x_B \in X$ closely parallels the one-dimensional treatment of McMurray (2017a). With quadratic utility, a voter prefers the platform closest to his expectation of z. Since his vote will only influence his utility if it is *pivotal* (event P), meaning that it makes or breakes a tie, he conditions on this event in addition to his private signal, as Lemma 1 now states.

[20]Symmetric voting is already implied by the assumption of Poisson population uncertainty (Myerson, 1998).

Lemma 1 *The voting strategy* v^{br} *is a best response to* $(v, x_A, x_B) \in V \times X^2$ *if and only if* $v^{br}(s) \in \arg\min_{j \in \{A,B\}} \|x_j - E(z|P, s)\|$ *for all* $s \in S$*.*

Since signals are informative of the truth, $E(z|s)$ is naturally monotonic in s. By itself, this does not guarantee that $E(z|P, s)$ is monotonic, given the intricate relationship between z and P, but Condition 4 ensures that $E(z|P, s)$ indeed is monotonic. In fact, it is linear in s, implying that the best response to any voting strategy is also *linear*, as defined in Definition 1. This means that voters with signals on one side of a line in S vote A, while voters with signals on the other side vote B.

Definition 1 $v_{h,c} \in V$ *is linear if* h *is a unit vector with polar angle* $\theta_h \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$ *and* $v(s) = \begin{cases} A & \text{if } h \cdot s < c \\ B & \text{if } h \cdot s > c \end{cases}$.[21] $v_{h,0} \equiv v_h$ *is a half-space strategy. A BNE* (v_h^*, x_A^*, x_B^*) *is a half-space equilibrium if* v_h^* *is a half-space strategy.*

Since the best response to a linear strategy is a linear strategy, and continuity is straightforward to verify, a standard fixed point argument over the compact set of lines in S guarantees equilibrium existence. To provide still greater symmetry, the analysis below restricts attention to *half-space* strategies, also defined in Definition 1, where the line that partitions S into regions of A and B voters passes through the origin.[22] Voters with signals in the general direction of a normal vector h vote for candidate B, where (without loss of generality) $\theta_h \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$ places B voters to the right of A voters in the horizontal dimension. In the analysis below, it is often convenient to use h and its orthogonal rotation $h' = R_{\frac{\pi}{2}} h$ as basis vectors. Lemma 2 now states that half-space strategies, which are monotonic and symmetric, produce electoral outcomes that are monotonic and symmetric, so that best response voting is monotonic and symmetric, as well.

[21] The behavior of voters for whom $h \cdot s = c$ exactly is inconsequential, occurring with zero probability.

[22] Section 5.3 explains that, in large elections, equilibria that differ substantially from half-space strategies cannot exist.

Lemma 2 *If $v_h \in V$ is a half-space strategy then, for any $z \in Z$,*

1. *(Outcome monotonicity)* $\nabla_z \Pr(B|z) \cdot h > 0$ *and* $\nabla_z \Pr(B|z) \cdot h' = 0$.

2. *(Pivot monotonicity)* $\nabla_z \Pr(P|z) \cdot h$ *and* $z \cdot h$ *have opposite signs and* $\nabla_z \Pr(P|z) \cdot h' = 0$.

3. *(Outcome symmetry)* $\Pr(A|-z) = \Pr(B|z)$ *and* $\Pr(A) = \Pr(B) = \frac{1}{2}$.

4. *(Pivot symmetry)* $\Pr(P|z) = \Pr(P|-z)$.

5. *(Half-space response) The unique best response to $(v_h, -x, x)$ is a half-space strategy $v_{h^{br}}$.*

When voters follow a half-space strategy, Part 1 of Lemma 2 states that electoral outcomes are monotonic: as z moves in the direction of h, signals in that vicinity become more likely, so more citizens vote B and victory becomes more likely. Part 2 states that pivot probabilities decline as z moves away from the origin. Moving z orthogonally does not change $\Pr(B)$ or $\Pr(P)$. Parts 3 and 4 state that opposite realizations of z generate opposite candidate fortunes but identical pivot probabilities. Part 5 states that, when candidates adopt symmetric platforms (as they will in equilibrium, given the outcome symmetry of Part 3), best-response voting follows another half-space strategy.

Like voters, a candidate prefers to implement her expectation of the optimal policy. With no private signals, both candidates' basic inclination is to adopt policy platforms at 0. A candidate's platform only matters if she wins office, however, so as I explain in McMurray (2018), she optimally restricts her attention to this event, updating her beliefs accordingly, in the same way that a voter restricts attention to the unlikely event of a pivotal vote. As Lemma 3 now states, candidates therefore condition their expectations on the event $w = j$. Note that this depends on the strategy that a candidate expects voters to follow, but not on the platform choice of her opponent.[23]

[23] The proof of Lemma 3 is a straightforward generalization of Theorem 2 in McMurray (2018), and so is not presented here. Note that this pivotal logic does not require the specific functional form of quadratic utility. With linear utility loss functions, for example, a candidate would prefer

Lemma 3 *For any voting strategy $v \in V$, the unique best response for candidate j is given by $x_j^{br} = E(z|j)$.*

When voters follow a half-space strategy, candidate A tends to win the election in certain states of the world while B wins in opposite states. From the event of winning, therefore, the two candidates infer opposite information and therefore adopt opposite platforms, as Lemma 4 now states, which will each be optimal in states of the world where they respectively win.

Lemma 4 *If $v_h \in V$ is a half-space strategy then $x_A^{br} = -x_B^{br} \neq 0$.*

In stating that candidates are symmetric, Lemma 3 says nothing about the extent of polarization. As I show in McMurray (2018), however, polarization can be substantial when n is large. This is because candidates A and B almost surely win when z is on opposite sides of a line in Z, so conditional on winning, candidate A is sure that z lies in one half-space while B is sure that it lies in the other.

If z_1 and z_2 are uncorrelated then $f(z)$ exhibits radial symmetry, as Section 3 notes, meaning that the optimal policy pair is equally likely to lie in any direction from the origin. The consequence of this, as Proposition 1 now states, is that *any* half-space strategy v_h, together with candidates' best response policies, constitutes an equilibrium. In such an equilibrium, candidates simply take policy positions in the directions of $-h$ and h, symmetric around the origin. Voters take the event of a pivotal vote into account, but behave just as they would if they did not: those with s_i closer to x_A vote A and those with s_i closer to x_B vote B.

Proposition 1 *Let $\rho = 0$. For any unit vector h there exists a unique half-space equilibrium (v_h^*, x_A^*, x_B^*), with $x_A^* = -x_B^* \neq 0$.*

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the median realization of z instead of the mean, but her posterior $f(z|w = j)$ would still condition on the event of winning the election.

The logic underlying Proposition 1 is straightforward: when voters follow v_h, electoral victory will be most likely when the optimal policy lies in the general directions of $-h$ and h, and perfect symmetry ensures that candidates form expectations precisely in these directions. A vote is most likely to be pivotal when z is roughly equidistant from $-h$ and h, and therefore roughly equidistant from x_A and x_B. This conveys nothing about which of the two platforms is superior, so a voter votes sincerely, as he would have done if he had not conditioned on the event of a pivotal vote.

Proposition 1 shows how a multidimensional environment reduces to a single dimension in equilibrium, with x_A and x_B endogenously defining "left" and "right" positions on the line between them, and voters dividing according to the projections of their opinions onto this line. So far, this has nothing to do with the specific structure of information, or even with common interests; it follows simply from having two candidates: *any* two positions in a multidimensional space define a line, and if each voter supports the candidate closest to himself (whether to his private interest or to his private estimate of the common interest) then voters will split into two groups, in the direction of that line.[24] However, showing how a *single* election reduces to one dimension does nothing to resolve the puzzle above, which is that issues are bundled together consistently *across* elections, so that different elections reduce to the *same* line. A unique equilibrium would have resolved the puzzle, by identifying a single orientation that must prevail in every election, but Proposition 1 states that equilibrium can be oriented in *any* direction. With perfect symmetry, then, indeterminacy is severe, and there is no obvious reason why different electorates could not bundle issues differently from one another.

Proposition 1 identifies infinitely many equilibria, but with only two policy dimensions, there are really only two ways to bundle the issues: any θ_h between 0 and

[24]With more than two candidates, voters should ignore all but the two front runners (Duverger, 1954), which would split the electorate in a similar way.

$\frac{\pi}{2}$ produces a *major* equilibrium, meaning that one candidate is more conservative on both issues than her opponent, while any θ_h between $-\frac{\pi}{2}$ and 0 produces a *minor* equilibrium, meaning that each candidate is more liberal on one issue and more conservative on the other. Within these categories, different θ_h merely correspond to different levels of polarization: for $|\theta_h| < \frac{\pi}{4}$, candidates polarize more on issue 1 than issue 2; for $|\theta_h| > \frac{\pi}{4}$, they polarize more on issue 2 than issue 1. In stating that any θ_h can sustain a half-space equilibrium, then, Proposition 1 implies *both* that either bundling of issues is possible in equilibrium, and that either issue can be more polarizing, with a continuum of possible polarization levels.

When $\rho = 0$, the distinction between major and minor equilibria is immaterial. When $\rho > 0$, however, major equilibria bundle issues in the direction of correlation while minor equilibria bundles issues oppositely. In that case, the number of equilibria falls precipitously, as Proposition 2 now states: there is a unique major equilibrium oriented exactly in the direction of the major diagonal, and a unique minor equilibrium oriented exactly in the direction of the minor diagonal.

Proposition 2 *If $\rho > 0$ then there exists one major half-space equilibrium $E^+ = \left(v_{h^+}, x_A^+, x_B^+\right)$ with $\theta_{h^+} = \theta_{x_B^+} = \frac{\pi}{4}$ and one minor half-space equilibrium $E^- = \left(v_{h^-}, x_A^-, x_B^-\right)$ with $\theta_{h^+} = \theta_{x_B^+} = -\frac{\pi}{4}$. No other half-space equilibrium exists.*

To give some intuition for why equilibrium half-space strategies can only be oriented in directions h^+ and h^-, Figure 2 illustrates the case of a half-space strategy with polar angle $\theta_h = 0$. When following this strategy, voters ignore s_2 completely: those with negative s_{i1} (unshaded region) vote A while those with positive s_{i1} (shaded region) vote B. Candidate B then infers that, if she wins the election, it will likely be because z_1 is positive. For $\rho = 0$, she would learn nothing about z_2, and would adopt a policy position exactly on the horizontal axis, but for $\rho > 0$ a positive z_1 suggests that z_2 is likely positive as well, so if she wins, candidate B takes positive positions on both issues—that is, adopting a platform with polar angle $\theta_{x_B^{br}} > 0$. Candidate A behaves symmetrically.

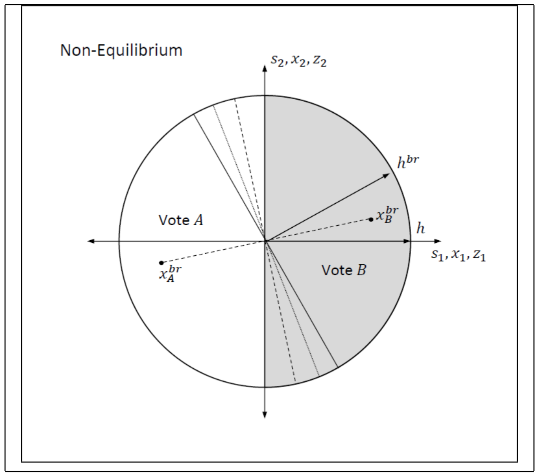

Figure 2: Non-equilibrium half-space strategy.

If candidates respond to v_h with platforms x_A^{br} and x_B^{br}, as illustrated in Figure 2, then the dashed line between them partitions Z such that voters with expectations $E(z|P,s)$ in the southwest and northeast regions prefer to vote A and B, respectively. Rotated counter-clockwise from the dashed line in Figure 2 is a dotted line. Voters whose expectations $E(z|s)$ lie southwest of the dotted line on the basis of private information alone form updated expectations $E(z|P,s)$ southwest of the dashed line to account for a pivotal vote, and therefore prefer to vote A; voters northeast of the dotted line update northeast of the dashed line, and prefer to vote B. These lines differ because, when they vote on the basis of s_1 alone, a voter's peers are most likely to tie (making his own vote pivotal) when z_1 is close to zero. Thus, for any s, $E(z|P,s)$ lies closer to the vertical axis than $E(z|s)$ does. In particular, if $E(z|s)$ lies exactly on the dotted line then $E(z|P,s)$ lies exactly on the dashed line, and a voter is indifferent between voting A and voting B.[25]

[25] A voter whose expectation $E(z|s)$ is northeast of the dotted line but southwest of the dashed line has a slightly negative signal of z_1 but a strongly positive signal of z_2. Since the two candidates are polarized largely only in the horizontal dimension, his basic inclination would be to vote for candidate A. If his vote is pivotal, however, it is likely that $z_1 \approx 0$. After conditinoning on event P, therefore, he puts relatively higher weight on his signal of z_2 than before, and votes for candidate B instead.

Corresponding to the dotted and dashed lines in Z is a solid line in S, also depicted in Figure 2. Voters with signal realizations southwest of this line form expectations $E(z|s)$ and $E(z|P,s)$ southwest of the dotted and dashed lines, respectively, and so prefer to vote A; symmetrically, voters northeast of this line prefer to vote B. In other words, if his peers follow v_h and candidates adopt best-response platforms x_A^{br} and x_B^{br}, then a voter's best response is the half-space strategy oriented in the direction of h^{br}, where $\theta_{h^{br}} > \theta_{x_B^{br}} > \theta_h$. Since v_h is not the best response to itself (and to the platforms that are candidates' best responses to v_h), it cannot be sustained in equilibrium.

The logic above is easiest to see for the case of $\theta_h = 0$, but holds more generally. As long as z_1 and z_2 are correlated, information that voters communicate about either dimension informs candidates about both dimensions. Half-space strategies with polar angles $\theta_h \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ communicate more about issue 1 than about issue 2; those with θ_h below $-\frac{\pi}{4}$ or above $\frac{\pi}{4}$ communicate more about issue 2 than about issue 1. Either way, taking the correlation across issues into account lessens the distinction between issues, so candidates' beliefs and platforms are less disproportionate, and closer to the main diagonal than h is. h^{br} must be closer still to the main diagonal, so that the voting behaviors assigned to each signal match candidates' positions even after pivotal considerations push voters' beliefs back in the direction of the original voting strategy.

If $\theta_h = \pm\frac{\pi}{4}$ then candidates infer equal information about the two issues, even after taking ρ into account, and adopt policy platforms exactly on the diagonal. Since this aligns perfectly with v_h, pivotality tells a voter nothing about which candidate is superior. h^{br} then coincides with h, and the voters who favor A or B after the pivotal voting calculus are the same ones who did so before taking P into account.

In the major equilibrium, votes for candidate B tend to reflect positive realizations of s_1, suggesting that z_1 is likely positive. They also tend to reflect positive s_2, suggesting that z_2 is likely positive, as well. Given the positive correlation between issues, inferring that $z_2 > 0$ makes candidate B more confident that $z_1 > 0$, and vice versa, so $E(z_1|B)$ and $E(z_2|B)$ are both more extreme than they would be if the truth variables were uncorrelated, for the same voting strategy. In the minor equilibrium, votes for candidate B tend to reflect positive realizations of s_1 but negative realizations of s_2, suggesting that $z_1 > 0 > z_2$. Since the issues are positively correlated, however, the inference that $z_2 > 0$ makes candidate B less confident that $z_1 < 0$, and vice versa, so $E(z_1|B)$ and $E(z_2|B)$ are less extreme than they would be if the truth variables were uncorrelated, for the same voting strategy. Proposition 3 states this formally, and notes further that the degree of polarization is monotonic in ρ.

Proposition 3 *$\|x_j^+\|$ and $\|x_j^-\|$ increase and decrease with ρ, respectively, and are equal if and only if $\rho = 0$.*

Private interest models highlight the utilitarian value of moderate policies, which compromise between the competing interests at either extreme to minimize the total disutility that voters suffer from policies far from their ideal points. From that perspective, Proposition 3 might seem to indicate that the minor equilibrium promotes greater social welfare. Centrist policies need not hold the same utilitarian appeal in common interest settings, however, because a voter benefits not from a policy that is close to his current opinion, but from a policy that is close to whatever is truly optimal. In any case, the result that there are two equilibria with differing levels of polarization raises the question of what is best for society. Defining social welfare $W(v, x_A, x_B)$ is uncontroversial here, unlike many settings, because voters and candidates share the same objective function, which can be written as follows.

$$W(v, x_A, x_B) = E_{w,z}[u(x_w, z)] = \int_Z \left[\sum_{j=A,B} u(x_j, z) \Pr(j|z) \right] f(z)\, dz \qquad (6)$$

Proposition 4 now states that, in fact, (6) is higher in the major equilibrium, even though policy outcomes are more extreme. Like polarization, the welfare difference between equilibria is monotonic in ρ.

Proposition 4 *$W(E^+;\rho)$ and $W(E^-;\rho)$ increase and decrease with ρ, respectively, and are equal if and only if $\rho = 0$.*

A simple intuition for the result that the major equilibrium is superior to the minor equilibrium is that v_{h^+} and v_{h^-} specify the same voter behavior, but in different states of the world. When z happens to be in quadrant 1 or quadrant 3, v_{h^+} does well at identifying the right quadrant, but v_{h^-} does not; similarly, v_{h^-} is effective at distinguishing between states of the world in quadrants 2 and 4, but v_{h^+} is not. Since quadrants 1 and 3 occur more frequently, v_{h^+} is the more informative voting strategy. In fact, it seems reasonable to conjecture that no other combination of voter and candidate behavior generates higher welfare than E^+.[26]

The existence of an inferior equilibrium implies that an inferior bundling of political issues could be self-perpetuating: even if it were known that issues had somehow come to be bundled together inefficiently, voters and candidates would go along with the inefficient bundling. If issues are only loosely correlated then little welfare is lost, but if ρ is large then the loss is more severe.

Proposition 2 reduces the number of equilibria from infinity to two, but since the two surviving equilibria entail opposite bundlings of the policy issues, it still gives no explanation as to why a major equilibrium should not prevail in one election while a minor equilibrium prevails in another. Proposition 4 is useful in that regard, in that a Pareto superior equilibrium survives the payoff dominance refinement of Harsanyi and Selten (1988). An even stronger reason to favor the major equilibrium is that the minor equilibrium is *unstable*, as the proof of Proposition 2 makes clear: rotating the voting strategy slightly away from v_{h^-} leads candidates to adopt platforms further from the minor diagonal, and voters respond by rotating further still, with a chain

[26] In common interest games such as this, behavior that is socially optimal is also individually optimal, and therefore constitutes an equilibrium (McLennan, 1998), so no half-space strategy other than v_{h^+} can maximize welfare, by Propositions 2 and 4. This do not rule out equilibria with asymmetric voting, but this seems unlikely to improve welfare.

of best responses converging to the major diagonal. In contrast, rotating slightly away from v_{h^+} generates best response platforms that rotate back again, prompting $v_{h^{br}}$ even closer to the major diagonal. Both for its efficiency and its stability, then, the major equilibrium emerges as the unique behavioral prediction of the model. Importantly, ρ need not be large: *any* positive correlation, no matter how small, uniquely pins down both the bundling of political issues and the extent of polarization, in fact prompting the same behavior that would prevail if ρ were equal to one.

The analysis above applies to finite electorates. This section considers electoral outcomes in the limit, as n grows large. To that end, let $E_n^+ = \left(v_{h_n^+}, x_{A,n}^+, x_{B,n}^+\right)$ and $E_n^- = \left(v_{h_n^+}, x_{A,n}^+, x_{B,n}^+\right)$ denote the major and minor equilibria for population size parameter n. Also, for any pair (x_A, x_B) of candidate platforms, partition Z into the sets $Z_A(x_A, x_B) = \{z \in Z : \|z - x_A\| < \|z - x_B\|\}$ of states that are closer to x_A than to x_B and its complement $Z_B(x_A, x_B) = \{z \in Z : \|z - x_A\| < \|z - x_B\|\}$. In particular, let $Z_j^+ = Z_j\left(x_{A,n}^+, x_{B,n}^+\right)$ and $Z_j^- = Z_j\left(x_{A,n}^-, x_{B,n}^-\right)$ denote the partitions that arise in equilibrium, which are the same for any n.

As the electorate grows large, the familiar logic of Condorcet's (1785) jury theorem guarantees that equilibrium voting satisfies *full information equivalence (FIE)*, meaning that the candidate whose platform is truly superior almost surely wins the election. Proposition 5 states this formally, along with the consequence that the event $w = j$ of winning the election converges to the event Z_j^+ or Z_j^- (depending on the equilibrium). Note that large elections do not remedy the welfare loss highlighted above: determining the sign of $z_1 + z_2$ is inherently more valuable than determining the sign of $z_1 - z_2$, so the major equilibrium remains more informative even in the limit.

Proposition 5 $\lim_{n\to\infty} \Pr\left(j|Z_j^+; E_n^+\right) = \lim_{n\to\infty} \Pr\left(j|Z_j^+; E_n^-\right) = 1$ *for* $j = A, B$. *Also,* $\lim_{n\to\infty} x_{j,n}^+ = E\left(z|Z_j^+\right)$ *and* $\lim_{n\to\infty} x_{j,n}^- = E\left(z|Z_j^-\right)$.

5 Extensions

5.1 Higher Dimensions

The model above takes the crucial first step of accommodating more than one dimension, but the eventual goal is to model a large number K of political issues. A thorough treatment of higher dimensions is beyond the scope of this paper, but this section discusses how, as long as symmetry is preserved, the two-dimensional analysis extends in a natural way to arbitrary K. To see this, let X be a K-dimensional unit hyperball with optimal issue positions $z_1, z_2, ..., z_K$ and suppose that the pairwise correlations between any two of these variables are the same, and proportional to $\rho \geq 0$. Assume further that $-z$ or permutations of z leave $f(z)$ unchanged (analogous to Condition 2) and that $\frac{\partial f(z)}{\partial z_k}$, $\frac{\partial^2 f(z)}{\partial z_k \partial \rho}$, and $\frac{\partial^2 f(z)}{\partial z_k \partial z_{k'}}$ have the same signs as $\rho \prod_{k' \neq k} z_{k'}$, $\prod_{k' \neq k} z_{k'}$, and ρ, respectively (as in Condition 1). Then let $g(s|z)$ satisfy linear informativeness, as already formulated in Condition 4.[27]

Formally extending the results of Section 4 would require the cumbersome notation of hyperspherical coordinates, but it should be clear from the analysis above that all of the results above have multidimensional analogs, based on identical reasoning. As in Lemma 1, each voter still favors the policy platform closest to his expectation $E(z|P, s)$ of the optimal policy, conditional on the event of a pivotal vote. Half-space strategies can still be defined by a single normal vector, now defining the hyperplane that partitions S, and such strategies still imply the symmetry properties of Lemma 2. A candidate's optimal platform choice is still her expectation $E(z|j)$ of the optimal policy, conditional on winning, and with a half-space voting strategy, this still implies that candidates will adopt substantially polarized platforms, opposite one another.

When $\rho = 0$, correlative monotonicity still implies that $f(z)$ is uniform. Together

[27] Examples of densities that satisfies these conditions are $\frac{1}{V_K}\left(1 + \rho \prod_{k=1}^{K} z_k\right)$, where V_K denotes the hypervolume of a K-dimensional unit hyperball (e.g. $f(z_1, z_2, z_3) = \frac{3}{4\pi}(1 + \rho z_1 z_2 z_3)$, in three dimensions), and $g(s|z) = \frac{1}{V_K}(1 + s \cdot z)$.

with the rotational symmetry of $g(s|z)$ and the symmetry of half-space voting, this guarantees that candidates' expectations $E(z|A)$ and $E(z|B)$ lie in the exact directions of $-h$ and h, respectively. If they adopt these expectations as platforms, a pivotal vote conveys nothing to voters about the magnitude of z in the direction of x_A and x_B, so $E(z|P,s)$ and $E(z|s)$ lie in the same direction, and voters with $s \cdot h < 0$ prefer to vote B, while those with $s \cdot h > 0$ prefer to vote A. In other words, v_h constitutes its own best response, for *any* normal vector h.

When $\rho > 0$, the number of equilibria reduces dramatically, as before. Voters cannot simply ignore all issues but the first, for example, for the same reason illustrated in Figure 2: if voting reflected s_1 alone, candidate B would infer upon winning that z_1 is likely positive, implying that z_2 through z_K are likely positive, as well, and would respond with positive positions on every issue. Relative to h, this reflects a rotation toward the major diagonal (i.e., where $z_k = z_{k'}$ for all k, k'). A voter's best response would then be a half-space strategy rotated even further toward the major diagonal.

Clearly, a major equilibrium still exists, with h^+, x_A^+, and x_B^+ all oriented along the major diagonal. In that equilibrium, a voter votes A if his average signal is negative and votes B if his average signal is positive. When candidate B wins the election, her updated expectations of z_k are then all (equally) positive. Given the positive correlation across issues, the inference that $z_k > 0$ reinforces the inference about the other issue dimensions, so that, as in Proposition 3, she adopts a more extreme position than she would have adopted if the issues had been uncorrelated. Candidate A takes an opposite position, and with the h^+, x_A^+, and x_B^+ exactly on the major diagonal, a pivotal vote conveys no information about which candidate is superior, thus sustaining the same voting strategy in response.

For three dimensions, Figure 3 illustrates the candidate platforms that best respond to the non-equilibrium half-space strategy described above, along with major and minor equilibrium platforms. In two dimensions there is only one minor equilib-

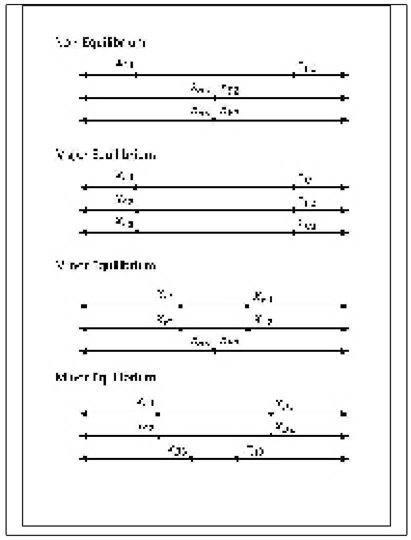

Figure 3: Equilibrium (and non-equilibrium) candidate positions in three dimensions with symmetric prior distribution.

rium, but $K = 3$ produces four of one type of minor equilibrium and three of another. The first possibility is that candidates polarize in opposite directions on two issues but not at all on the third. Maintaining the assumption that $x_{A1} \leq x_{B1}$, there are four such equilibria, because candidates can converge on any issue, and converging on issue 1 gives two ways to polarize on issues 2 and 3. In the first minor equilibrium of Figure 3, for example, $x_{A3} = x_{B3} = 0$. In response, voters ignore s_3 completely, voting A if $s_1 < s_2$ and voting B if $s_1 > s_2$. Upon winning, candidate B then expects z_1 to be positive and z_2 to be negative. These inferences undermine one another, so she is less extreme than she would otherwise be. The inferences that $z_1 > 0$ and $z_2 < 0$ have opposite implications for z_3, which cancel out in equilibrium so that $E(z_3|j) = 0$.

In the second type of minor equilibrium, one candidate takes a negative position on two issues and a positive position on one, while the other does the opposite. There are three such equilibria, orienting any of the three issues opposite the other two. In the final example of Figure 3, for example, $x_{A1} < x_{B1}$ and $x_{A2} < x_{B2}$ but $x_{A3} > x_{B3}$. Upon winning, candidate B then infers that $z_1 > 0$ and $z_2 > 0$ but that $z_3 < 0$. The inference about z_1 and z_2 undermines the inference about z_3, so she takes a position on issue 3 that is less extreme than it would be if issues were uncorrelated. The inference that $z_1 > 0$ is similarly undermined by the inference that $z_3 < 0$, but is bolstered by the inference that $z_2 > 0$, so she adopts a more extreme position on issue 1 than on issue 3. Issues 1 and 2 are symmetric, so she takes equally extreme positions.

In higher dimensions, the number of minor equilibria grows quickly. In four dimensions, for example, there are sixteen minor equilibria: three with both candidates adopting leftist positions on two issues and rightist positions on two issues; four with one candidate taking leftist positions on three issues and a rightist position on the fourth issue or vice versa; and nine with each candidate taking a leftist position on one issue, a rightist position on one issue, and centrist positions on the remaining two issues.[28] For arbitrary K, there are enough minor equilibria for a candidate to take leftist or rightist positions on *any* strict subset of the issues. In other words, *any* bundling of issues could persist in equilibrium, with polarization depending on how many issues are bundled in each direction.

For large K, coordinating on any one minor equilibrium seems difficult. In contrast, there is only ever a single major equilibrium, so coordination is easy. This equilibrium also Pareto dominates the others, and so survives the payoff dominance refinement (Harsanyi and Selten, 1988). Moreover, the minor equilibria are all unstable. For example, perturbing the first minor equilibrium of Figure 3 so that candidates polarize slightly more on issue 1 than issue 2 would lead voters to place greater weight on s_1 than s_2, thus conveying more information about z_1 than about z_2, so that candidates polarize even more on issue 1 and even less on issue 2 in response. Information about z_3 would then no longer cancel out, so $E(z_3|A) < 0 < E(z_3|B)$

[28]This accounting of equilibria retains the convention above that the candidate who is weakly to the right on issue 1 is labeled as candidate B. Dropping this convention, there are two minor equilibria in two dimensions, twelve in three dimensions, and twenty-six in four dimensions.

and therefore $x_A^{br} < 0 < x_B^{br}$. As candidates polarize more on issues 1 and 3 and less on issue 2, $v_{h^{br}}$ rotates further. As A and B votes increasingly convey information that z_1 and z_3 are positive, platforms on issue 2 become less and less polarized, until they are not polarized at all, and then polarize in the opposite direction, consistent with issues 1 and 3. As before, this chain of best responses converges to the major equilibrium.

5.2 Asymmetric Issue Importance

To keep the pivotal voting calculus tractable, Section 3 assumes that X and Z are symmetric in every direction, and so is f when $\rho = 0$ (Condition 3); even when $\rho > 0$, f is symmetric along both diagonals (Condition 2); g exhibits both rotational symmetry and error symmetry (Conditions 5 and 6); utility u weights directions and issues symmetrically; half-space equilibria ensure symmetric platforms; and simultaneous timing preserves symmetric voting even when candidates deviate asymmetrically. Perfect symmetry is worrisome, because it is a knife-edge condition and because various asymmetries seem entirely plausible. If reducing the number of symmetric directions from infinity to two reduces the number of equilibria similarly, relaxing symmetry further might eliminate remaining equilibria.

This section makes progress on this question by relaxing symmetry in a way that preserves one form of symmetry, while still preserving the half-space structure of equilibrium voting that makes analysis tractable. Consider the following generalized utility function,

$$u(x, z) = -(1 + \lambda)(x_1 - z_1)^2 - (1 - \lambda)(x_2 - z_2)^2$$

where the model of Section 3 imposes $\lambda = 0$, but $\lambda \in (0, 1)$ allows the possibility that issue 1 is more important to voters than issue 2.[29] Dropping terms that don't depend

[29] Or, isomorphically, that that deviations from the status quo are easier in one direction than the other, so that the effective policy space is an ellipse, not a circle.

on the policy outcome, expected utility then generalizes from (2) to the following.

$$E_z\left[u\left(x,z\right)|\Omega\right] = -\left(1+\lambda\right)\left[x_1 - E\left(z_1|\Omega\right)\right]^2 - \left(1-\lambda\right)\left[x_2 - E\left(z_2|\Omega\right)\right]^2 \tag{7}$$

Proposition 6 now characterizes equilibrium for $\rho = 0$ and $\lambda > 0$. Like correlation across issues, asymmetric issue importance eliminates all of the infinitely many equilibria identified in Proposition 1, except two. One of these focuses entirely on issue 1 ($\theta_{h^+} = 0$) and the other focuses entirely on issue 2 ($\theta_{h^-} = -\frac{\pi}{2}$). Clearly, the first of these provides higher welfare, since issue 1 is more important.

Proposition 6 *If $\rho = 0$ and $\lambda > 0$ then there exists one half-space equilibrium with $\theta_{h^*} = x_B^* = 0$ and another with $\theta_{h^*} = x_B^* = \frac{\pi}{2}$. No other half-space equilibrium exists.*

The logic of Proposition 6 is that candidate incentives do not depend on λ: each simply prefers to implement her expectations of z_1 and z_2, regardless of which is more important. If voters followed a half-space strategy with $\theta_h = \frac{\pi}{4}$, for example, candidates would still respond with platforms on the major diagonal. However, λ does affect voter responses. For example, a voter with s_i on the minor diagonal was previously indifferent between candidates on the major diagonal, as each was better on one issue. Now, since issue 1 is more important, such a voter responds with a half-space strategy oriented clockwise from h^+. In general, voters with the new utility function value policy vector x just as voters with the old utility function valued $\tilde{x} = \begin{pmatrix} 1+\lambda \\ 1-\lambda \end{pmatrix} \cdot x$. As in the case of $\rho > 0$, this drives a wedge between v_h and the best response to candidates' best responses to v_h, except when voters and candidates exactly align with one of the major axes, so that x_j and $\tilde{x}_j$ lie in the same direction.

When λ and ρ are both zero, the model above is symmetric in every direction, and there are half-space equilibria oriented in every direction. When either parameter is positive, the model is symmetric only in two directions, and only two half-space equilibria remain. If λ and ρ are both positive then the model is no longer symmetric in any direction, which raises the question of whether equilibrium exists at all.

The answer, as Proposition 7 now states, is that a major equilibrium and a minor equilibrium remain. These are no longer oriented exactly along the major and minor diagonals, but are in the same vicinity.

Proposition 7 *If* $\lambda > 0$ *and* $\rho > 0$ *then there exists a major equilibrium* $\left(v_{h^+}, x_A^+, x_B^+\right)$ *with* $\theta_{h^+} \in \left(0, \frac{\pi}{4}\right)$ *and a minor equilibrium* $\left(v_{h^-}, x_A^-, x_B^-\right)$ *with* $\theta_{h^-} \in \left(-\frac{\pi}{4}, -\frac{\pi}{2}\right)$.

The proof of Proposition 7 notes that if a voter's peers follow a half-space strategy with $\theta_h = 0$ then, because $\rho > 0$, candidates respond with platforms rotated counter-clockwise, to $\theta_{x_B^{br}} > 0$, rotating the best voting voting strategy counter-clockwise from h. If $\theta_h = \frac{\pi}{4}$ then $\theta_{x_B^{br}} = \frac{\pi}{4}$, as well, but since $\lambda > 0$, a voter's best response is rotated clockwise from h. By continuity, there is a polar angle $\theta_{h^+} \in \left(0, \frac{\pi}{4}\right)$ that prompts the same polar angle in response. By similar arguments, half-space strategies with polar angles $\theta_h = -\frac{\pi}{4}$ and $\theta_h = -\frac{\pi}{2}$ generate best-response half space strategies with larger and smaller polar angles, so continuity implies the existence of an equilibrium oriented toward $\theta_{h^-} \in \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right)$.

Proposition 7 makes clear that Proposition 2 is robust even though Proposition 7 is not: in spite of asymmetry, one major equilibrium and one minor equilibrium still exist. It would be straightforward to adapt the proof of Proposition 4 to show that the major equilibrium Pareto dominates, both because it bundles issues more efficiently and because it focuses more squarely on the more important of the two issues. The minor equilibrium offers little differentiation on the more important issue, focusing more heavily on what is less important, in addition to bundling issues inefficiently. The major equilibrium is also stable, as a half-space strategy close to h^+ generates a best-response vector even closer to h^+, while the minor equilibrium is unstable, as a half-space strategy close to h^- generates a sequence of best responses that converges to h^+.

The equilibria of Proposition 7 are no longer oriented exactly along the major and minor diagonals. However, $x_{Ak} < x_{Bk}$ for both k still, so this represents the same bundling of issues as before. That they are off the diagonals simply means that polarization differs across issues. In other words, λ determines which of the two issues is more polarized, but ρ still determines how the issues are bundled in equilibrium.

5.3 General Asymmetry and Nonlinearity

Relaxing all of the symmetry and linearity of Section 3, equilibrium exists as long as X is non-empty and compact and S is measurable. To see this, note that in common interest environments, socially optimal behavior constitutes an equilibrium (McLennan, 1998). Fixing (x_A, x_B), the correspondence of welfare-maximizing voting strategies $v_n^{**}(x_A, x_B)$ is non-empty, convex, compact, and upper hemicontinuous in (x_A, x_B) for any n, by the maximum theorem.[30] Lemma 3 still gives candidates' unique best responses $x_{A,n}^{br}(v) = E(z|A;n,v)$ and $x_{B,n}^{br}(v) = E(z|B;n,v)$ to any voting strategy, so v_n^{**} can be reinterpreted as a correspondence from the compact set V of voting strategies into itself (i.e., the optimal response to candidates' best responses to any voting strategy). For any n, Kakutani's theorem guarantees a fixed point v_n^*, and $\left(v_n^*, x_{A,n}^{br}(v_n^*), x_{B,n}^{br}(v_n^*)\right)$ therefore constitutes an equilibrum.

Intuitively, it seems that voting in the optimal equilibrium—or, for that matter, best-response voting in *any* equilibrium—should be monotonic, meaning that voters with signals closest to x_j vote j (making j's victory most likely when z is in the same vicinity), even if the boundary in S between A voters and B voters is nonlinear or no longer passes through the origin. Unfortunately, the intricate relationship between z, P, s, and w makes these conjectures difficult to verify.

As long as utility is monotonic in $\|x - z\|$, A and B should optimally win in the

[30]Measurable S implies that Lebesgue integration, and therefore welfare, are continuous on the set V of measurable (mixed) voting strategies $v : S \to [0,1]$, which is compact under the product topology (by Tychanoff's theorem) since X is compact. Convexity of $v^{**}(x_A, x_B)$ follows because welfare is monotonic (and therefore quasiconcave) in v.

sets Z_A and Z_B of states closer to x_A and to x_B. As long as Z_A and Z_B generate distinguishable distributions of signals, Barelli, Bhattacharya, and Siga (2018) show that a sequence of voting strategies $(v_n)_n$ exists that satisfies full information equivalence in the limit.[31] Since $v_n^*(x_A, x_B)$ produces (weakly) greater welfare than v_n, $(v_n^*(x_A, x_B))_n$ then satisfies FIE as well. Moreover, since $V \times X^2$ is compact, a subsequence of $\left(v_n^*, x_{A,n}^{br}(v_n^*), x_{B,n}^{br}(v_n^*)\right)_n$ converges to some $\left(v_\infty^*, x_{A,\infty}^*, x_{B,\infty}^*\right)$, and the continuity of $x_{j,n}^{br}(v)$ guarantees that $x_{j,\infty}^* = E\left(z|z \in Z_j\left(x_{A,\infty}^*, x_{B,\infty}^*\right)\right)$. In other words, candidates behave in large elections as if they were playing a simpler game, where they choose platforms and then the platform closer to z is implemented.

One general consequence of this is substantial polarization in large elections, as platforms correspond to expectations $E(z|Z_A)$ and $E(z|Z_B)$ over opposite sides of a partition. Another general consequence is (approximate) endogenous symmetry within the policy space, in that the unconditional mean $E(z)$ tends to be located centrally within the policy space, and is a weighted average of the two platforms.

Without assuming specific functional forms, it seems inherently difficult to characterize the role of ρ explicitly. However the rest of the model is specified, though, it seems intuitively that correlation across issues should operate as above. In particular,

[31] More precisely, there exists $(v_n)_n$ satisfying FIE, if and only if a hyperplane can partition the space $\Delta(S)$ of signal distributions such that $\{g(s|z) \in \Delta(S) : z \in Z_A\}$ and $\{g(s|z) \in \Delta(S) : z \in Z_B\}$ lie on opposite sides of the partition. Above, Conditions 2 and 4 guarantee that, interpreting the set of functions on S as a vector space, $H(s) = (x_B - x_A) \cdot s$ is normal to such a hyperplane. To see this, note that $g(s|z = \bar{x})$ must lie on the hyperplane of indifference, and when s is uniform, the difference

$$\begin{aligned}
\langle g(s|z) - g(s|\bar{x}), H(s) \rangle &= \int_S [(z - \bar{x}) \cdot s] [(x_B - x_A) \cdot s] \, ds \\
&= (z - \bar{x}) \cdot \begin{pmatrix} (x_{B1} - x_{A1}) \int_S s_1^2 ds + (x_{B1} - x_{A1}) \int_S s_1 s_2 ds \\ (x_{B1} - x_{A1}) \int_S s_1 s_2 ds + (x_{B1} - x_{A1}) \int_S s_2^2 ds \end{pmatrix} \\
&= (z - \bar{x}) \cdot (x_B - x_A) \int_S s_1^2 ds
\end{aligned}$$

is proportional to the utility difference $u(x_B, z) - u(x_A, z) = -(x_B - z) \cdot (x_B - z) + (x_A - z) \cdot (x_A - z) = 2(x_B - x_A) \cdot (z - \bar{x})$.

starting from any equilibrium where $\rho = 0$ and $x_A = E(z|Z_A)$ and $x_B = E(z|Z_B)$, increasing ρ amounts to raising the density of (z_1, z_2) pairs with the same sign and lowering the density of pairs with opposite signs, which will tend to rotate $E(z|Z_A)$ and $E(z|Z_B)$ in the direction of the major diagonal. In response, candidates should rotate in this direction, and Z_A and Z_B should rotate with them, so that $E(z|Z_A)$ and $E(z|Z_B)$ rotate even further. Without perfect symmetry, of course, equilibrium platforms will not lie exactly on the major diagonal (see Section 5.2), but should generally lie in the same quadrant (or orthant, in higher dimensions), thus generating the same bundling of issues. Formally, the maximum theorem implies that $\left(v_n^*, x_{A,n}^{br}(v_n^*), x_{B,n}^{br}(v_n^*)\right)$ is upper hemicontinuous in u, X, Z, S, f, and g, implying that slight deviations from the case of perfect symmetry produce equilibria that deviate only slightly from the equilibrium characterized above.

Starting from the case of $\rho = 0$, platforms can rotate toward the major diagonal both in clockwise or counter-clockwise directions. In that light, the logic above also suggests the possibily general existence of a minor equilibrium, delicately balanced between rotating in either direction. As before, such an equilibrium should be unstable, in the sense that rotating slightly away rotates best responses even further, in contrast with the major equilibrium, where rotations in either direction should trigger best responses that rotate back again.

To illustrate this numerically, consider $X = [-1, 1]^2$ and the density $f(z_1, z_2) = \frac{1}{8}(1 + \rho z_1 z_2) + \frac{1}{8}(1 - z_1)$, which is not symmetric in any direction when $\rho > 0$. For $\rho = 0$, there are four policy pairs satisfying $x_j = E(z|j)$. These are oriented vertically $\left(\binom{-.17}{-.50}, \binom{-.17}{.50}\right)$, horizontally $\left(\binom{-.58}{0}, \binom{.39}{0}\right)$, southwest/northeast $\left(\binom{-.50}{-.27}, \binom{.21}{.31}\right)$, and southeast/northwest $\left(\binom{-.50}{.27}, \binom{.21}{-.31}\right)$. Increasing ρ (which is proportional to the correlation coefficient) slightly to .1, these rotate so that three $\left(\binom{-.19}{-.50}, \binom{-.15}{.50}\right)$, $\left(\binom{-.57}{-.03}, \binom{.39}{.04}\right)$, and $\left(\binom{-.51}{-.27}, \binom{.22}{.31}\right)$ are southwest/northeast but one $\left(\binom{-.50}{.28}, \binom{.20}{-.30}\right)$ is still southeast/northwest. The latter seems to be the most fragile, and produces the lowest welfare.

5.4 Timing, Office Motivation, and Candidate Information

Section 3 assumes that candidates and voters move simultaneously but, in reality, candidates announce policy positions before voting takes place. Timing does not matter to voters, who best respond to candidates' platforms either way, but knowing that her policy choice will change how voters vote does matter to a candidate. The analysis of the previous section is useful in that regard: in large elections (as long as FIE is satisfied), equilibrium behavior is equivalent to the simpler game in which candidates move first and then voters, observing candidates' positions, determine which platform is superior.[32] In that sense, equilibrium behavior in simultaneous and sequential games are asymptotically equivalent.

In Section 3, candidates care only about the policy outcome. In reality, candidates might also value winning office. With simultaneous timing, candidates have no way to influence voters, but since timing is immaterial in large elections, insights from one-dimensional sequential game in McMurray (2018) should apply. As that paper shows, polarization is lower when candidates are office motivated, but can still be substantial, because truth motivated voters only sometimes reward moderation.[33] Formally extending to multiple dimensions would be difficult for the reasons described above, but electoral concerns should give a candidate no reason to rotate her policy position, at least in a symmetric environment like the above, as doing so would attract votes in some states of the world but sacrifice votes in opposite states, which are equally likely.

As noted above, another unrealistic assumption of Section 3 is that candidates have no private information about z. With sequential timing, however, this is actually immaterial, as I explain in McMurray (2018). As long as her signal is informative of the truth, a candidate's platform must be monotonic in her signal; with sequen-

[32] Equilibrium exists in a sequential game, for reasons similar to those above.

[33] Moderating her platform always attracts votes, but this only matters to a candidate for moderate realizations of z: when z is extreme in her (or her opponent's) favor, a candidate will win (or lose) whether she moderates or not.

tial timing, this means that voters can infer candidates' private signals, effectively updating the common prior. Voters then adjust their behavior accordingy, and in equilibrium elect the candidate whose platform is superior *after* taking this information into account. A candidate's optimal platform is still her expectation $E(z|j)$ conditional on winning, but the event $w = j$ now incorporates both her own information and her opponent's.[34] In the limit as n grows large, this still converges to $\mathbf{1}_{u(x_j,z)>u(x_{-j},z)}$, so $E(z|j)$ still converges to $E(z|Z_j)$, as before.[35]

6 Applications

As Sections 1 and 2 explain, private interest literature cannot adequately explain why political attitudes are so unidimensional. As Shor (2014) expresses, for example, "it is not clear why environmentalism necessarily hangs together with a desire for more union prerogatives, but it does." In a common interest setting, such correlation arises naturally from the logical connections between issues: for example, environmentalism and union support might both reflect a view of businesses as ruthless, willing to pursue profit at the expense of employees or the environment. In fact, such a view could also engender support for minimum wage laws and a host of other pro-labor policies. Such logical connections may seem too weak to justify such consistent issue bundling, but *any* non-zero ρ is sufficient to orient the equilibrium, so that issues are bundled just as they would be if correlation were perfect. As issue importance fluctuates, canddiates polarize most highly on the prominent issue of the day, but the

[34] Modeling this explicitly would involve higher order beliefs that seem hopelessly intractable. For example, the informational content of voting behavior now mixes original private information, responses to a candidate's own information (which she finds redundant), responses to her opponent's original information (which she also hopes to infer), responses to her opponent's guess of her own information, and so on.

[35] Similar reasoning would apply with subjective prior beliefs about z.

underlying bundling of issues remains largely the same.[36]

Converse (1964) and Shor (2014) find that political candidates are more ideologically consistent than voters. This, too, is consistent with the analysis above. With two issues, for example, voters hold opinions in every quadrant, but candidates only ever take positions in quadrants 1 and 3. For large K, candidates adopt consistent positions on every issue, but voters form opinions in every orthant, and the fraction of voters who favor the party line on every issue tends to zero.

The results above also shed light on modern political arguments. The U.S. Libertarian party, for example, is liberal on social issues such as immigration, abortion and marriage, but conservative on economic issues such as taxes and regulation. Its website emphasizes logical consistency, arguing that while Democrats and Republicans each favor personal or economic liberty, Libertarians favor both.[37] The model above formalizes this as a claim that the optimal social and economic policies should be correlated, implying that society is stuck in an inferior equilibrium. The analysis above affirms this as a possibility, although it also concludes that such an equilibrium is unlikely to prevail. One possibility is simply that correlation goes the other way, and the present bundling of issues is efficient: like their names suggest, for example, conservative policies might be logically unified by a commitment to preserve social and economic traditions, while liberal or progressive policies seek to modernize on both fronts. On the other hand, enriching the present model might vindicate the Libertarian narrative: like z, for example, ρ may be imperfectly observed, with different voters seeing it as positive or negative. The point here is not to settle any philosophical debate, but to show that the model clarifies current public debate.

[36] Policy realignments do seem to take place occasionally. One possibility is that this reflects learning about ρ. With new insights about the relationship between issues, for example, a correlation that had long been presumed positive might prove to be negative. In that case, what had seemed to be a stable, major equilibrium would suddenly be revealed as a minor equilibrium, giving way to a (rather sudden) rebundling of issues.

[37] See www.lp.org/platform.

7 Conclusion

Multidimensional election models are plagued by convergence or equilibrium non-existence or multiplicity. Empirical unidimensionality makes this problem seem less urgent, but remains its own mystery. This paper has pointed out that, in a common interest setting, logical connections across issues are a natural source of correlation, and breaks symmetry to reduce the potential of multiple equilibria. The possibility remains of an inferior bundling of issues, mirroring prevalent public concerns, but the only stable equilibrium efficiently bundles related issues together. Decisions that are inherently multidimensional thus endogenously reduce to a single, "left-right" axis, and a fixed logical structure explains why the direction of disagreement remains so consistent over space and time.

That a common interest paradigm sheds light where private interest literature has not highlights the utility of this general approach to elections. Literally identical preferences are improbable, however, so generalizing this is an important direction for future work.[38] As long as preferences share a substantial common element, the results above seem likely to be robust. If a voter's ideal policy were some weighted average of the policy $\hat{x}_i$ that maximizes his narrow self-interest and the social optimum z, for instance, he should still formulate an expectation $E(z|P, s_i)$ of z based on his private signal, together with whatever he can infer from the event of a pivotal vote. As in the model above, correlation between z_1 and z_2 should lead voters to develop correlated expectations. As long as private interests are not too dominant, voting should then reveal information about voters' private signals, which a truth motivated candidate can use in much the same way as before.[39] Candidates who mix private and public interest should still condition their beliefs about z on the

[38] Even if private interests are important empirically, a pure common interest model is useful as a theoretical benchmark against which imperfectly aligned interests can be compared.

[39] If voting is only partly informative then candidates will polarize less, for a given n, but may be just as polarized in the limit, as they approach perfect information.

event of winning, and if voter strategies reveal information in a particular direction then $E(z|j)$ should tend to rotate from there, toward the major diagonal, as before. Sufficiently selfish candidates (with particular policy preferences) might divide the electorate southeast/northwest, but those who sufficiently value the public good should divide it southwest/northeast.[40]

Within the pure common interest paradigm, future work should enrich the information structure above. In the current framework, for example, pairs of voters could reach a consensus, simply by sharing and combining their private signals. In large groups, public opinion should be so reliable that voters abandon minority opinions. Empirically, individuals routinely disagree with one another, and maintain unpopular opinions. In McMurray (2018) I discuss informational limitations that might explain these features, and the same discussion applies here.[41] In higher dimensions, it would be useful to explore correlation structures that lack the symmetry assumed in Section 5.1. With three dimensions, for example, it is possible for z_1 and z_2 to correlated positively with each other, but negatively with z_3. A dynamic model of how opinions update between elections would be useful, as well: Krasa and Polborn (2014) present evidence, for example, that the correlation of voter attitudes across political issues has increased over time.

Condorcet's (1785) jury theorem ensures that voters elect the candidate with the better platform. In one dimension, this means that the policy outcome x is generally in the same direction as z. In higher dimensions, x is only guaranteed to be in the right half-space; as the number of dimensions grows large, the probability of being in

[40] Even candidates who are inherently very selfish may put substantial weight on the public interest, to secure a favorable legacy.

[41] Conflicts of interest are a more obvious barrier to consensus, but similar disagreements persist on purely speculative questions, where interests should be irrelevant. Another relevant observation is that, in learning from other voters' opinions, an individual must discount any information that is already shared. Identifying which pieces of information another voter shares would require extensive communication, and disagreements could persist in the meantime.

the same orthant as z shrinks to zero, and the expected number of issues on which x and z differ grows without bound. This is an inherent limitation of a binary decision in a highly complex policy environment. Note that additional candidates cannot easily solve this problem, as they reduce the best candidate's ability to win a plurality of votes. If two candidates close to z split voters' support, for example, someone far inferior may win. Precisely to avoid such situations, strategic voters might also ignore all but two candidates, as Duverger's (1954) law predicts.

In the one-dimensional model of McMurray (2017b), the winning candidate infers a policy mandate from the size of her vote share, which shapes her policy choices and can even steer her precisely to z. Candidates who lose the election then still add value, by giving voters a way to signal more or less extreme opinions. With multiple dimensions, losing candidates may play an even more important role, allowing voters to signal opinions orthogonal to the winner's platform. Whether Democrat and Republican platforms reflect a major or a minor equilibrium, for example, votes for Libertarian or Green candidates could nudge a major party to increase freedom or environmental protections. With far more issues than parties, however, the ability to precisely identify z remains limited, underscoring the importance of informal political activities such as petitions, rallies, public opinion surveys, and letters to legislators, which communicate policy-specific opinions that a coarse voting mechanism cannot.[42]

A Appendix

Proof of Lemma 1. Given $z \in Z$ and $v \in V$, a voter votes for candidate $j \in \{A, B\}$ with probability

$$\phi(j|z) = \int_S 1_{v(s)=j}\, g(s|z)\, ds \tag{8}$$

[42]Making a similar observation, Besley and Coate (2008) advocate un-bundling complex legislation, allowing separate dimensions to be decided separately.

where $1_{v(s)=j}$ equals one if $v(s) = j$ and zero otherwise. The numbers N_A and N_B of A and B votes are then independent Poisson random variables with means $n\phi(A|z)$ and $n\phi(B|z)$, with the following probability of vote totals $(N_A, N_B) = (a, b)$.

$$\psi(a, b|z) = \frac{e^{-n}}{a!b!} [n\phi(A|z)]^a [n\phi(B|z)]^b \tag{9}$$

From within the game, an individual reinterprets N_A and N_B as the numbers of votes cast by his peers (Myerson, 1998); by voting himself, he can add one to either total. A vote for j will be *pivotal* (event P_j) with probability $\Pr(P_j|z) = \frac{1}{2}\Pr(N_j = N_{-j}) + \frac{1}{2}\Pr(N_j = N_j - 1)$, because candidates tie but j loses the tie-breaking coin toss or j wins the coin toss but loses the election by exactly one vote. The total probability of a vote for either candidate being pivotal (event P) is then given by $\Pr(P|z) = \Pr(P_A|z) + \Pr(P_B|z)$.

In terms of $\Pr(P_j)$ and $\Pr(P)$, the difference in expected benefit between voting B and voting A is given by the following,

$$\begin{aligned}
\Delta E_{w,z}[u(x_w, z)|s] &= \int_Z [u(x_B, z) - u(x_A, z)] \Pr(P_B|z) f(z|s)\,dz \\
&\quad - \int_Z [u(x_A, z) - u(x_B, z)] \Pr(P_A|z) f(z|s)\,dz \\
&= \int_Z [u(x_B, z) - u(x_A, z)] \Pr(P|z) f(z|s)\,dz \\
&= \Pr(P|s) E_z[u(x_B, z) - u(x_A, z)|P, s] \\
&= \Pr(P|s)\left(-\|x_B - E(z|P, s)\|^2 + \|x_A - E(z|P, s)\|^2\right)
\end{aligned} \tag{10}$$

where the third equality follows because $f(z|s) = \frac{f(z)g(s|z)}{\int_Z g(s|z)f(z)dz}$, $f(z|P, s) = \frac{\Pr(P|z)f(z)g(s|z)}{\int_Z \Pr(P|z)g(s|z)f(z)dz}$, and $\Pr(P|s) = \frac{\int_Z \Pr(P|z)g(s|z)f(z)dz}{\int_Z g(s|z)f(z)dz}$, so $\Pr(P|z) f(z|s) = \Pr(P|s) f(z|P, s)$, and the final equality follows from (2). This benefit is positive if and only if x_B is closer to $E(z|P, s)$ than x_A is. ∎

Proof of Lemma 2. 1. With a half-space strategy, (8) reduces to $\phi(A|z) = \int_{\{s\in S: s\cdot h<0\}} g(s|z)\,ds$ and $\phi(B|z) = \int_{\{s\in S: s\cdot h>0\}} g(s|z)\,ds$. These decrease and increase with z in the direction of h: for example, $\nabla_z \phi(B|z) \cdot h = \int_{\{s \in S: s \cdot h > 0\}} g_1 (s \cdot h)\, ds > 0$. In the orthogonal direction, both are constant: using basis vectors h and h', for example, $\nabla_z \phi(B|z) \cdot h' = \int_{\{s \in S: s_1 > 0\}} g_1 s_2 ds = g_1 \int_{\{s \in S: s_1 > 0, s_2 > 0\}} [s_2 + (-s_2)]\, ds = 0$.

Conditional on k total votes (and on z), the number of B votes follows a binomial distribution, with probability parameter $\phi(B|z)$. $\Pr(B|k, z)$ is the probability that this number exceeds $\frac{k}{2}$, which increases with $\phi(B|z)$. Summing over all k, this implies that $\Pr(B|z)$ increases in $\phi(B|z)$, too. $\Pr(B|z)$ does not otherwise depend on z, so $\nabla_z \phi(B|z) \cdot h > 0$ implies that $\nabla_z \Pr(B|z) \cdot h = \frac{\partial \Pr(B|z)}{\partial \phi(B|z)} \nabla_z \phi(B|z) \cdot h > 0$ and, similarly, $\nabla_z \phi(B|z) \cdot h' = 0$ implies that $\nabla_z \Pr(B|z) \cdot h' = 0$.

2. $\Pr(P|z)$ increases in $\phi(B|z)$ only if $\phi(B|z) < \frac{1}{2}$. To see this, rewrite $\Pr(P)$ in terms of $\phi(B|z)$ using (9) and $\Pr(P_j)$, as follows.

$$
\begin{aligned}
\Pr(P|z) &= \sum_{k=m}^{\infty} \left[\psi(k, k|z) + \frac{1}{2}\psi(k+1, k|z) + \frac{1}{2}\psi(k, k+1|z) \right] \\
&= \sum_{k=m}^{\infty} \frac{n^{2k} e^{-n}}{k!k!} [\phi(A|z)\, \phi(B|z)]^k \left[1 + \frac{n\phi(B|z)}{2(k+1)} + \frac{n\phi(A|z)}{2(k+1)} \right] \\
&= \sum_{k=m}^{\infty} \frac{n^{2k} e^{-n}}{k!k!} \{[1 - \phi(B|z)]\, \phi(B|z)\}^k \left[1 + \frac{n}{2(k+1)} \right] \qquad (11)
\end{aligned}
$$

This expression increases in $[1 - \phi(B|z)]\, \phi(B|z)$, which increases in $\phi(B|z)$ if and only if $\phi(B|z) < \frac{1}{2}$. By Part 1 of this lemma, $\phi(B|z = 0) = \frac{1}{2}$, and since $\nabla_z \phi(B|z) \cdot h' = 0$, this implies further that $\phi(B|z) = \frac{1}{2}$ whenever $z \cdot h = 0$. Since $\nabla_z \phi(B|z) \cdot h > 0$, this implies that $\Pr(P|z)$ increases in $\phi(B|z)$ when $z \cdot h < 0$ and decreases in $\phi(B|z)$ when $z \cdot h > 0$. Accordingly, $\nabla_z \Pr(P|z) \cdot h = \frac{\partial \Pr(P|z)}{\partial \phi(B|z)} \nabla_z \phi(B|z) \cdot h$ and $z \cdot h$ have opposite signs, while $\nabla_z \Pr(P|z) \cdot h' = \frac{\partial \Pr(P|z)}{\partial \phi(B|z)} \nabla_z \phi(B|z) \cdot h' = 0$.

3. If v_h is a half-space strategy then $\phi(A| - z) = \int_{\{s \in S: s \cdot h < 0\}} g(-s|z)\, ds = \int_{\{s \in S: s \cdot h > 0\}} g(s|z)\, ds = \phi(B|z)$, where the first equality utilizes the rotational symmetry of g (Condition 5) and the second reflects a change of variables. From (9) it is then clear that $\psi(a, b| - z) = \psi(b, a|z)$ for any $a, b \in Z_+$, implying that $\Pr(A| - z) = \sum_{k=0}^{\infty} \sum_{m=1}^{\infty} \psi(k + m, k| - z) + \frac{1}{2} \Pr(N_A = N_B)$ equals $\Pr(B|z) = \sum_{k=0}^{\infty} \sum_{m=1}^{\infty} \psi(k, k + m|z) + \frac{1}{2} \Pr(N_A = N_B)$ and, integrating over z, that $\Pr(A) =$

$\Pr(B) = \frac{1}{2}$.

4. $\phi(A|-z) = \phi(B|z)$ implies that $\Pr(P|-z) = \Pr(P|z)$, as is clear from (11).

5. If $x_A = -x_B$ then the difference (10) in utility between voting B and voting A can be written as $\left[-(x_{B1}-z_1)^2 - (x_{B2}-z_2)^2\right] - \left[-(-x_{B1}-z_1)^2 - (-x_{B2}-z_2)^2\right] = 4(x_{B1}z_1 + x_{B2}z_2)$ in state z. Averaging across states, this equals

$$\begin{aligned}\Delta E_{w,z}\left[u(x_w)|s\right] &= \frac{4}{g_0}\int_Z (x_{B1}z_1 + x_{B2}z_2)\Pr(P|z)\left[g_0 + g_1(s\cdot z)\right]f(z)\,dz \\ &= \frac{4g_1}{g_0}\int_Z (x_B\cdot z)\Pr(P|z)(s\cdot z)f(z)\,dz\end{aligned}$$

where the final equality follows from Condition 2. This expression is linear in s and equals zero for $s=0$, implying that $\Delta E_{w,z}\left[u(x_w)|s\right]$ is positive and negative on opposite sides of a line in S that passes through the origin. In other words, the best response is another half-space strategy, as claimed. ∎

Proof of Lemma 4. Lemma 3 states that $x_j^{br} = E(z|j)$ for $j = A, B$, and symmetry is a straightforward consequence of Condition 2, together with Part 3 of Lemma 2, since $x_{Ak}^{br} = \int_Z z_k \frac{\Pr(A|z)f(z)}{\Pr(A)}dz = \int_Z (-z_k)\frac{\Pr(A|-z)f(-z)}{\Pr(A)}dz$ equals $-\int_Z z_k \frac{\Pr(B|z)f(z)}{\Pr(B)}dz = -x_{Bk}^{br}$. Using h and h' as basis vectors, $E(z|B)\cdot h$ can be written simply as $E(z_1|B)$. Part 1 of Lemma 2 implies that $\Pr(B|z_1,z_2)$ increases in z_1 and is constant with respect to z_2, so that this reduces to the following, and can seen to be positive.

$$\begin{aligned}E(z_1|B) &= \int_Z z_1 \frac{\Pr(B|z_1,z_2)f(z_1,z_2)}{\Pr(B)}dz_2dz_1 \\ &= 2\int_{Z_{1,2}} [z_1\Pr(B|z_1,z_2)f(z_1,z_2) - z_1\Pr(B|-z_1,z_2)f - z_1, z_2 \\ &\quad + z_1\Pr(B|z_1,-z_2)f(z_1,-z_2) - z_1\Pr(B|-z_1,-z_2)f(-z_1,-z_2)]dz_2dz_1 \\ &= 2\int_{Z_{1,2}} z_1\left[\Pr(B|z_1,z_2) - \Pr(B|-z_1,z_2)\right]\left[f(z_1,z_2) + f(z_1,-z_2)\right]dz_2dz_1 \qquad (12)\end{aligned}$$

Here, $Z_{1,2}$ denotes the union of the first and second octants (i.e., the first quadrant) and the second equality holds because $\Pr(B) = \frac{1}{2}$, by Lemma 2. ∎

Proof of Proposition 1. By Lemmas 3 and 4, candidates' best responses to v_h are symmetric expectations $x_A^{br} = E(z|A) = -E(z|B) = -x_B^{br}$. When $\rho = 0$, $x_B^{br} \cdot h' = 0$, meaning that $\theta_{x_B^{br}} = \theta_h$. To see this, write x_B^{br} using h and h' as basis vectors, so that $x_B^{br} \cdot h$ and $x_B^{br} \cdot h'$ reduce simply to $E(z_1|B)$ and $E(z_2|B)$, respectively. The first of these is given by (12) in the proof of Lemma 4, and the second reduces in a similar way to the following.

$$E(z_2|B) = 2\int_{Z_{1,2}} z_2 \left[\Pr(B|z_1, z_2) - \Pr(B| - z_1, z_2)\right]\left[f(z_1, z_2) - f(z_1, -z_2)\right] dz_2 dz_1 \tag{13}$$

When $\rho = 0$, Condition 3 implies that $f(z_1, -z_2) = f(z_1, z_2)$, so (13) equals zero. Thus, x_B^{br}—and, by symmetry, x_A^{br}—are orthogonal to h'. As shown in the proof of Lemma 4, however, $x_B^{br} \cdot h > 0$. Thus, x_A^{br} and x_B^{br} lie exactly in the directions of $-h$ and h, respectively.

By Lemma 2, a voter's best response to v_h and symmetric candidate platforms $(-x, x)$ is another half-space strategy, $v_{h^{br}}$. By Proposition 1, a voter prefers to vote B if and only if $E(z|P, s) \cdot x_B > 0$, and since $\theta_{x_B} = \theta_h$ in equilibrium, this is equivalent to the condition that $E(z|P, s) \cdot h > 0$ (where P depends implicitly on v_h). With basis vectors h and h', this dot product reduces simply to $E(z_1|P, s)$, which is proportional to the following.

$$\begin{aligned}
&\int_{Z_{1,2}} [z_1 \Pr(P|z_1, z_2)\, g(s|z_1, z_2)\, f(z_1, z_2) - z_1 \Pr(P| - z_1, z_2)\, g(s| - z_1, z_2)\, f(-z_1, z_2) \\
&+ z_1 \Pr(P|z_1, -z_2)\, g(s|z_1, -z_2)\, f(z_1, -z_2) - z_1 \Pr(P| - z_1, -z_2)\, g(s| - z_1, -z_2)\, f(-z_1, -z_2)] dz_2 dz_1 \\
= &\int_{Z_{1,2}} z_1 \Pr(P|z_1, z_2)\, [[g(s|z_1, z_2) - g(s| - z_1, -z_2)]\, f(z_1, z_2) \\
&+ [g(s|z_1, -z_2) - g(s| - z_1, z_2)]\, f(z_1, -z_2)] dz_2 dz_1
\end{aligned} \tag{14}$$

Equality follows from Part 2 of Lemma 2 since (with the rotated basis) $\Pr(P|z_1, z_2)$ is constant with respect to z_2, and depends on the magnitude of z_1 but not the sign. Condition 3 implies that $f(-z_1, z_2) = f(z_1, z_2)$ and Condition 4 implies that $g(s|z_1, z_2) - g(s| - z_1, z_2)$ has the same sign as $\binom{s_1}{s_2} \cdot \binom{z_1}{z_2} - \binom{s_1}{s_2} \cdot \binom{z_1}{z_2} = 2s_1 z_1$. Since z_1 is positive on $Z_{1,2}$, this has the same sign as s_1, or to $s \cdot h$ for the original Euclidean basis vectors. In other words, v_h is the best response to itself, and together with the best-response candidate platforms constitutes a half-space equilibrium. ∎

Lemma A1 *If* $\theta_h \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ *and* $\theta_z \in \left(\theta_h, \theta_h + \frac{\pi}{2}\right)$ *then* $f(z) - f(M_{\theta_h} z)$ *has the same sign as* $\rho\left(\frac{\pi}{4} - |\theta_h|\right)$.

Proof. If $|\theta_h| \leq \frac{\pi}{4}$ then $\theta_z - \theta_h \in \left(0, \frac{\pi}{2}\right)$ implies that $\max\left\{\theta_{M_{\theta_h} z}, \theta_{M_{-\frac{\pi}{4}} M_{\theta_h} z}\right\} < \min\left\{\theta_z, \theta_{M_{\frac{\pi}{4}} z}\right\}$. By Condition 2, both vectors on the left-hand side of this inequality have density $f(M_{\theta_h} z)$ and both vectors on the right have density $f(z)$. Moreover, both sides of the inquality lie in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$, so by Condition 1, $f(z) - f(M_{\theta_h z})$ has the same sign as ρ, as claimed. Similar reasoning applies for other values of θ_h: $\theta_z - \theta_h \in \left(0, \frac{\pi}{2}\right)$ implies that $-\frac{\pi}{4} < \max\left\{\theta_{R_{-\pi} z}, \theta_{M_{-\frac{\pi}{4}} R_{-\pi} z}\right\} < \min\left\{\theta_{M_{\theta_h} z}, \theta_{M_{\frac{\pi}{4}} M_{\theta_h} z}\right\} < \frac{\pi}{4}$ if $\theta_h > \frac{\pi}{4}$ and implies that $-\frac{\pi}{4} < \max\left\{\theta_z, \theta_{M_{-\frac{\pi}{4}} z}\right\} < \min\left\{\theta_{R_\pi M_{\theta_h} z}, \theta_{M_{\frac{\pi}{4}} R_\pi M_{\theta_h} z}\right\} < \frac{\pi}{4}$ if $\theta_h < -\frac{\pi}{4}$. Either way, both vectors on the left have density $f(z)$ and both vectors on the right have density $f(M_{\theta_h z})$, so Condition 1 implies that $f(z) - f(M_{\theta_h z})$ and ρ have opposite signs. ∎

Proof of Proposition 2. If his peers vote according to v_h then, according to Lemma 1, a voter's best response is to vote j if and only if $E(z|P,s) \cdot x_j > 0$. For v_h to be its own best response, voters with signals satisfying $s \cdot h > 0$ should have a best response to vote B while the rest vote A. A signal orthogonal to h should make a voter indifferent. According to Lemma 3, candidate j's best response to v_h is the policy $x_j^{br} = E(z|j)$ that is optimal in expectation, conditional on winning. Taking these conditions together, a half-space equilibrium requires that $E(z|P,s) \cdot E(z|j)$ have the same sign as $s \cdot h$. The logic of this proof is to show that this is possible if and only if $\theta_h \in \left\{-\frac{1}{4}, \frac{1}{4}\right\}$. In particular, other values of θ_h produce $E(z|P,s) \cdot E(z|j) \neq 0$ for s orthogonal to h.

The cleanest way to compare $E(z|P,s)$ and $E(z|j)$ with each other is to compare both vectors with h and h'. This is accomplished most simply by using h and h' as basis vectors, so that $x_j^{br} \cdot h$ and $x_j^{br} \cdot h'$ reduce to $E(z_1|B)$ and $E(z_2|B)$, respectively, which are given by (12) and (13). Similarly, $E(z|P,s) \cdot h$ and $E(z|P,s) \cdot h'$ reduce to $E(z_1|P,s)$ and $E(z_2|P,s)$. For generic s, the first of these is given in the proof of Proposition 1 as proportional to (14). Analogously, the second is as follows.

$$\begin{aligned} E(z_2|P,s) \;\; \propto \;\; & \int_{Z_{1,2}} z_2 \Pr(P|z_1,z_2) \left[\left[g(s|z_1,z_2) - g(s|-z_1,-z_2) \right] f(z_1,z_2) \right. \\ & \left. + \left[g(s|-z_1,z_2) - g(s|z_1,-z_2) \right] f(z_1,-z_2) \right] dz_2 dz_1 \end{aligned} \tag{15}$$

However, a signal that is orthogonal to h has $s_1 = 0$; in that case, Condition 6 implies that $g(s|-z_1,z_2) = g(s|z_1,z_2)$ for any z_1, so (14) and (15) reduce to the following.

$$\begin{aligned} E(z_1|P,s=h') \;\; \propto \;\; & \int_{Z_{1,2}} z_1 \Pr(P|z_1,z_2) \left[g(s|z_1,z_2) - g(s|z_1,-z_2) \right] \\ & \times \left[f(z_1,z_2) - f(z_1,-z_2) \right] dz_2 dz_1 \end{aligned} \tag{16}$$

$$\begin{aligned} E(z_2|P,s=h') \;\; \propto \;\; & \int_{Z_{1,2}} z_2 \Pr(P|z_1,z_2) \left[g(s|z_1,z_2) - g(s|z_1,-z_2) \right] \\ & \times \left[f(z_1,z_2) + f(z_1,-z_2) \right] dz_2 dz_1 \end{aligned} \tag{17}$$

Together, $s = h'$ and $z \in Z_{1,2}$ also imply that $g(s|z_1,z_2) > g(s|z_1,-z_2)$.

If $|\theta_h| < \frac{\pi}{4}$ then, given $\rho > 0$, Lemma A1 implies that $f(z) > f(M_{\theta_h} z)$. In terms of basis vectors h and h', this means that $f(z_1,z_2) > f(z_1,-z_2)$, implying that (12), (13), (16), and (17) are all positive. That $x_B^{br} \cdot h > 0$ and $x_B^{br} \cdot h' > 0$ implies that x_B^{br} has polar angle strictly between those of h and h'.[43] That $E(z|P,s) \cdot h > 0$ and $E(z|P,s) \cdot h' > 0$, together with the result that x_B^{br} lies between h and h', imply in turn that $E(z|P,s) \cdot x_B^{br} > 0$, as well.[44] In short, $|\theta_h| < \frac{\pi}{4}$ is not compatible with equilibrium: when his peers follow v_h and candidates respond optimally, equilibrium would require that a voter with orthogonal signal $s = h'$ be indifferent between voting A and voting B, but instead such a voter prefers to vote B.

[43]Formally, $x_B^{br} \cdot h = r_{x_B^{br}} \cos\left(\theta_{x_B^{br}} - \theta_h\right) > 0$ and $x_B^{br} \cdot h' = r_{x_B^{br}} \cos\left(\theta_{x_B^{br}} - \theta_{h'}\right) > 0$ together imply that $\theta_{x_B^{br}} - \theta_h < \frac{\pi}{4}$ and $\theta_{h'} - \theta_{x_B^{br}} < \frac{\pi}{4}$.

[44]Formally, $x_B^{br} = r_{x_B^{br}} \left[\alpha h + (1-\alpha) h'\right]$ for some $\alpha \in (0,1)$ implies that $E(z|\mathcal{P},s) \cdot x_B^{br} = r_{x_B^{br}} \alpha E(z|\mathcal{P},s) \cdot h + r_{x_B^{br}} (1-\alpha) E(z|\mathcal{P},s) \cdot h' > 0$.

If $|\theta_h| > \frac{\pi}{4}$ then Lemma A1 implies that $f(z) < f(M_{\theta_h} z)$, which in terms of basis vectors h and h' means that $f(z_1, z_2) > f(z_1, -z_2)$. In that case, (12) is still positive but (13) is now negative, meaning that $x_B^{br} \cdot h' < 0 < x_B^{br} \cdot h$, so that x_B^{br} has polar angle strictly between those of $-h'$ and h. Similarly, for $s = h'$ (17) is still positive but (16) is negative, meaning that $E(z|P, s) \cdot x_B^{br} < 0$.[45] When his peers follow v_h and candidates respond optimally, therefore, a voter with signal $s = h'$ prefers to vote A. Thus, v_h is incompatible with equilibrium.

If $|\theta_h| = \frac{\pi}{4}$ then Lemma A1 implies that $f(z) = f(M_{\theta_h} z)$, which in terms of basis vectors h and h' means that $f(z_1, z_2) = f(z_1, -z_2)$. In that case, (12) and (16) are still positive but (13) and (17) are zero. That $x_B^{br} \cdot h > 0$ and $x_B^{br} \cdot h' = 0$ implies that x_B^{br} is colinear with h. That $E(z|P, s) \cdot h > 0$ and $E(z|P, s) \cdot h' = 0$ when $s = h'$ then implies that the best response to v_h (and $x_A^{br}(v_h)$ and $x_B^{br}(v_h)$) is v_h, so $\left(v_h, x_A^{br}, x_B^{br}\right)$ constitutes a half-space equilibrium. ■

Proof of Proposition 3. Since x_B^+ lies in the direction of h^+ and x_B^- lies in the direction of h^-, their magnitudes can be written as the projection of x_B^+ on h^+ and the projection of x_B^- on h^-, respectively. Generically, (12) gives the projection of x_B^{br} on h, in terms of the basis vectors h and h'. That candidate platforms in the major and minor equilibria are symmetric implies that $\Pr(B|-z) = \Pr(A|z)$, while $|\theta_h| = \frac{\pi}{4}$ implies that $f(M_{\theta_h} z) = f(z)$ by Lemma A1. Thus, (12) reduces to $\|x_B\| = 8 \int_{Z_{1,2}} z_1 \left[\Pr(B|z_1, z_2) - \frac{1}{2}\right] f(z_1, z_2)\, dz$, and can be differentiated as follows.

$$\frac{\partial \|x_B\|}{\partial \rho} = 8 \int_{Z_{1,2}} z_1 \left[\Pr(B|z_1, z_2) - \frac{1}{2}\right] \frac{\partial}{\partial \rho} f(z_1, z_2)\, dz$$

A policy vector z under these rotated basis vectors corresponds to a rotated vector $R_{\theta_h} z$ under the original basis vectors. In the cases of the major and minor equilibria, this corresponds to $R_{\frac{\pi}{4}} z = \frac{1}{\sqrt{2}} \binom{z_1 - z_2}{z_1 + z_2}$ and $R_{-\frac{\pi}{4}} z = \frac{1}{\sqrt{2}} \binom{z_1 + z_2}{z_2 - z_1}$, respectively. Reversing z_1 and z_2 therefore corresponds to reversing the sign of either the first or the second

[45] Formally, $x_B^{br} = r_{x_B^{br}} [\alpha h + (1 - \alpha)(-h')]$ for some $\alpha \in (0, 1)$ and $E(z|\mathcal{P}, s) \cdot h < 0 < E(z|\mathcal{P}, s) \cdot h'$ together imply that $E(z|\mathcal{P}, s) \cdot x_B^{br} = r_{x_B^{br}} \alpha E(z|\mathcal{P}, s) \cdot h - r_{x_B^{br}} (1 - \alpha) E(z|\mathcal{P}, s) \cdot h' < 0$.

component. Either way, because of the dimensional symmetry of f (Condition 2), this is equivalent to reversing the sign of ρ. Thus, $\frac{\partial\|x_B\|}{\partial\rho}$ reduces further, as follows,

$$\begin{aligned}\frac{\partial\|x_B\|}{\partial\rho} &= 8\int_{Z_1}\{z_1\left[\Pr(B|z_1,z_2)-\frac{1}{2}\right]\frac{\partial}{\partial\rho}f(z_1,z_2)\\&\quad+z_2\left[\Pr(B|z_2,z_1)-\frac{1}{2}\right]\frac{\partial}{\partial\rho}f(z_2,z_1)\}dz\\&= 8\int_{Z_1}\{z_1\left[\Pr(B|z_1,z_2)-\frac{1}{2}\right]\\&\quad-z_2\left[\Pr(B|z_2,z_1)-\frac{1}{2}\right]\}\frac{\partial}{\partial\rho}f(z_1,z_2)\,dz\end{aligned}$$

where $z_1 > z_2 > 0$ for policy pairs in Z_1, implying that $\Pr(B|z_1,z_2) > \Pr(B|z_2,z_1)$, and therefore that the bracketed difference is positive. With the original basis vectors, Condition 1 states that $\frac{\partial}{\partial\rho}f(z)$ has the same sign as z_1z_2; with the rotated basis vectors, this means that $\frac{\partial}{\partial\rho}f(z)$ has the same sign as $(z_1-z_2)(z_1+z_2)$ for the major equilibrium and $(z_1+z_2)(z_2-z_1)$ for the minor equilibrium. Since $z_1 > z_2 > 0$ for all policy pairs in Z_1, this implies that $\frac{\partial\|x_B^+\|}{\partial\rho}$ is positive and $\frac{\partial\|x_B^-\|}{\partial\rho}$ is negative, as claimed. ■

Proof of Proposition 4. If voter and candidate strategies $(v,x_A,x_B)=(v_+,(-x,-x),(x,x))$ are oriented along the major diagonal (for some $x>0$) then (6) can be rewritten as follows

$$\begin{aligned}W_+(x) &= \int_Z\left\{\left[-(-x-z_1)^2-(-x-z_2)^2\right]\Pr(A|z)+\left[-(x-z_1)^2-(x-z_2)^2\right]\Pr(B|z)\right\}f(z)\,dz\\&= 2\int_Z\left[-(x-z_1)^2-(x-z_2)^2\right]\Pr(B|z)\,f(z)\,dz\end{aligned}$$

using a change of variables and noting that $\Pr(A|z)=\Pr(B|-z)$ by Lemma 2. This function is concave in x, achieving a maximum at the major equilibrium policy position $x_+=E(z_1|B)=E(z_2|B)$. Implicitly, $\Pr(B|z)$ and $f(z)$ in this expression depend on v_+ and ρ, respectively.

Instead orienting voter and candidate strategies $(v,x_A,x_B)=(v_-,(-x,x),(x,-x))$ along the minor diagonal (for some $x>0$) generates the same welfare as reversing the sign of ρ,

$$\begin{aligned}
W_-(x;\rho) &= \int_Z \{[u((-x,x),(z_1,z_2))\Pr(A|z_1,z_2;v_-) \\
&\quad + u((x,-x),(z_1,z_2))\Pr(B|z_1,z_2;v_-)]\} f(z_1,z_2;\rho)\,dz \\
&= \int_Z \{[u((x,x),(-z_1,z_2))\Pr(B|-z_1,z_2;v_+) \\
&\quad + u((x,x),(z_1,-z_2))\Pr(B|z_1,-z_2;v_+)]\} f(z_1,z_2;\rho)\,dz \\
&= \int_Z \{u((x,x),(z_1,z_2))\Pr(B|z_1,z_2;v_+) \\
&\quad + u((x,x),(z_1,z_2))\Pr(B|z_1,z_2;v_+)\} f(z_1,z_2;-\rho)\,dz \\
&= W_+(x;-\rho)
\end{aligned}$$

where the second equality holds because $u((-x,x),(z_1,z_2)) = -(-x-z_1)^2 - (x-z_2)^2$ and $u((x,x),(-z_1,z_2)) = -(x+(-z_1))^2 - (x-z_2)^2$ are equivalent algebraically (as are $u((x,-x),(z_1,z_2))$ and $u((x,x),(z_1,-z_2))$), because $\Pr(A|z) = \Pr(B|-z)$, and because expected vote shares $\phi(B|z_1,z_2;v_-) = \int_{s\cdot\binom{x}{-x}>0} g(s|z_1,z_2)\,ds$ and $\phi(B|z_1,-z_2;v_+) = \int_{s\cdot\binom{x}{x}>0} g(s|z_1,-z_2)\,ds$ are equivalent (as can be seen by a change of variables in the second dimension), so $\Pr(B|z_1,z_2;v_-) = \Pr(B|z_1,-z_2;v_+)$.

An alternative way of rewriting (6) is as follows.

$$\begin{aligned}
W_+(x) &= \int_Z [u(x_A,z)\Pr(A|z) + u(x_B,z)\Pr(B|z)]\, f(z)\,dz \\
&= 2\int_{Z_{1,2,3,8}} [u(x_A,z)\Pr(A|z) + u(x_B,z)\Pr(B|z)]\, f(z)\,dz \\
&= 4\int_{Z_{1,8}} [u(x_A,z)\Pr(A|z) + u(x_B,z)\Pr(B|z)]\, f(z)\,dz \\
&= 4\int_{Z_1} (\{[-(-x-z_1)^2 - (-x-z_2)^2]\Pr(A|z_1,z_2) \\
&\quad + [-(x-z_1)^2 - (x-z_2)^2]\Pr(B|z_1,z_2)\} f(z_1,z_2) \\
&\quad + \{[-(-x-z_1)^2 - (-x+z_2)^2]\Pr(A|z_1,-z_2) \\
&\quad + [-(x-z_1)^2 - (x+z_2)^2]\Pr(B|z_1,-z_2)\} f(z_1,-z_2))dz \qquad (18)
\end{aligned}$$

The second equality here holds because $x_A = -x_B$, so $u(x_A,z)\Pr(A|z) = u(x_B,-z)\Pr(B|-z)$ and $u(x_B,z)\Pr(B|z) = u(x_A,-z)\Pr(A|-z)$, implying that each realization of z in orthants 1, 2, 3, or 8 generates identical welfare to the opposite state, $-z$, from orthant 4, 5, 6, or 7. The third equality holds because $u((-x,-x),(z_2,z_1))$ and $u((-x,-x),(z_1,z_2))$ are algebraically equivalent (as are $u((-x,-x),(z_2,z_1))$ and $u((x,x),(z_1,z_2)))$ and because the expected vote shares $\phi(B|z_2,z_1) = \int_{s\cdot\binom{x}{x}>0} g(s|z_2,z_1)\,ds$ and $\phi(B|z_1,z_2) = \int_{s\cdot\binom{x}{x}>0} g(s|z_1,z_2)\,ds$ in states (z_2,z_1) and (z_1,z_2) are the same, so $\Pr(j|z_2,z_1) = \Pr(j|z_1,z_2)$ for $j = A, B$. The final equality associates with each policy pair (z_1,z_2) in Z_1 a corresponding policy pair $(z_1,-z_2)$ in Z_8.

$f(-z_1,z_2;\rho) = f(z_1,z_2;-\rho)$ by Condition 2, so differentiating (18) with respect to ρ and replacing $\Pr(A|z_1,z_2) = 1-\Pr(B|z_1,z_2)$ yields the following.

$$\begin{aligned}
\frac{\partial W_+(x;\rho)}{\partial\rho} &= 4\int_{Z_1}\{\left[-(-x-z_1)^2-(-x-z_2)^2\right][1-\Pr(B|z_1,z_2)] \\
&\quad +\left[-(x-z_1)^2-(x-z_2)^2\right]\Pr(B|z_1,z_2) \\
&\quad -\left[-(-x-z_1)^2-(-x+z_2)^2\right][1-\Pr(B|z_1,-z_2)] \\
&\quad -\left[-(x-z_1)^2-(x+z_2)^2\right]\Pr(B|z_1,-z_2)\}\frac{\partial}{\partial\rho}f(z_1,z_2)\,dz \\
&= 4\int_{Z_1}\{-4xz_2+(4xz_1+4xz_2)\Pr(B|z_1,z_2) \\
&\quad +(-4xz_1+4xz_2)\Pr(B|z_1,-z_2)\}\frac{\partial}{\partial\rho}f(z_1,z_2)\,dz \\
&= 16x\int_{Z_1}\{z_1[\Pr(B|z_1,z_2)-\Pr(B|z_1,-z_2)] \\
&\quad +z_2\left[\Pr(B|z_1,z_2)-\frac{1}{2}+\Pr(B|z_1,-z_2)-\frac{1}{2}\right]\}\frac{\partial}{\partial\rho}f(z_1,z_2)\,dz
\end{aligned}$$

$(z_1,z_2)\in Z_1$ (equivalently, $z_1>z_2>0$) implies: that $\Pr(B|z_1,z_2)>\frac{1}{2}$ and $\Pr(B|-z_1,z_2)>\frac{1}{2}$, since $\binom{z_1}{z_2} = \frac{z_2-z_1}{2}\binom{-1}{1}+\frac{z_1+z_2}{2}\binom{1}{1}$ and $\binom{z_1}{-z_2} = -\frac{z_1+z_2}{2}\binom{-1}{1}+\frac{z_1-z_2}{2}\binom{1}{1}$, so Parts 1 and 3 of Lemma 2 imply that $\Pr(B|0,0)=\frac{1}{2}$ and that $\nabla_z\Pr(B|z)\cdot\binom{z_1}{z_2} = \frac{z_1+z_2}{2}\nabla_z\Pr(B|z)\cdot h_+>0$ and $\nabla_z\Pr(B|z)\cdot\binom{z_1}{-z_2} = \frac{z_1-z_2}{2}\nabla_z\Pr(B|z)\cdot h_+>0$; that $\Pr(B|z_1,z_2)>\Pr(B|z_1,-z_2)$, since $\binom{z_1}{z_2}-\binom{z_1}{-z_2} = \frac{z_1+2z_2}{2}\binom{-1}{1}+z_2\binom{1}{1}$, so $\nabla_z\Pr(B|z)\cdot\left[\binom{z_1}{z_2}-\binom{z_1}{-z_2}\right] = z_2\nabla_z\Pr(B|z)\cdot h_+>0$; and that $\frac{\partial}{\partial\rho}f(z_1,z_2)>0$. Thus, $W_+(x;\rho)$ increases in ρ (strictly, as long as $x>0$). If (x_+^ρ,x_+^ρ) and $(x_-^\rho,-x_-^\rho)$ denote the major and minor equilibrium platforms of candidate B, therefore, then for $\rho' > \rho$, $W_+\left(x_+^{\rho'};\rho'\right) \geq W_+\left(x_-^{\rho};\rho'\right) \geq W_+\left(x_-^{\rho};\rho\right)$. That is, major equilibrium welfare increases in ρ. Minor equilibrium welfare decreases in ρ, since $W_-\left(x_-^{\rho'};\rho'\right) = W_+\left(x_-^{\rho'};-\rho'\right) < W_+\left(x_-^{\rho};-\rho\right) < W_+\left(x_-^{-\rho};-\rho\right) = W_-\left(x_-^{-\rho};\rho\right) < W_-\left(x_-^{\rho};\rho\right)$, implying that $W_+\left(x_+^{\rho};\rho\right) = W_-\left(x_-^{\rho};\rho\right) = W_+\left(x;-\rho\right)$ if and only if $\rho = 0$, as claimed. ■

Proof of Proposition 6. Regardless of λ, Lemmas 3 and 2 remain valid: best response platforms are given by $x_j^{br} = E\left(z|j\right)$ and half-space voting v_h produces expected vote shares that increase in the direction of h, symmetric platforms $x_A^{br} = -x_B^{br}$, and a half-space best-response strategy $v_{h^{br}}$. The logic for the last of these claims mirrors the proof of Lemma 2: the difference in expected utility between voting B and voting A now becomes the following,

$$\begin{aligned}\Delta E_{w,z}\left[u\left(x_w\right)|s\right] &= \int_Z \{\left[-\left(1+\lambda\right)\left(x_{B1}-z_1\right)^2 - \left(1-\lambda\right)\left(x_{B2}-z_2\right)^2\right] \\ &\quad -\left[-\left(1+\lambda\right)\left(-x_{B1}-z_1\right)^2 - \left(1-\lambda\right)\left(-x_{B2}-z_2\right)^2\right]\} \Pr\left(P|z\right) f\left(z|s\right) dz \\ &= 4\int_Z \left[\left(1+\lambda\right)x_{B1}z_1 + \left(1-\lambda\right)x_{B2}z_2\right]\left[1+\frac{g_1}{g_0}\left(s_1z_1+s_2z_2\right)\right] f\left(z\right) dz\end{aligned}$$

which is still linear in s and still zero for $\left(s_1,s_2\right) = \left(0,0\right)$.

From (7), the benefit of voting B has the same sign as $\left(1+\lambda\right)x_{B1}E\left(z_1|P,s\right) + \left(1-\lambda\right)x_{B2}E\left(z_2|P,s\right) = E\left(z|P,s\right)\cdot\binom{(1+\lambda)x_{B1}}{(1-\lambda)x_{B2}}$. The proof of Proposition 1 shows that if $\rho = 0$ then a voter with $\theta_s = \theta_h + \frac{\pi}{2}$ is indifferent between voting A and B. That is, $E\left(z|P,s\right)\cdot\binom{x_{B1}}{x_{B2}} = 0$. Such a voter is no longer indifferent when $\lambda > 0$, unless $x_{B1} = 0$ or $x_{B2} = 0$. That proof shows further that x_B^{br} has the same polar angle as h, however, so equilibrium requires $h_1 = 0$ or $h_2 = 0$, meaning that $\theta_h \in \left\{0,\frac{\pi}{2}\right\}$. ■

Lemma A2 *If voting follows v_h then: if $\theta_h \in \left[-\frac{\pi}{4},\frac{\pi}{4}\right]$ then $\theta_{x_B^{br}} \in \left[\theta_h,\frac{\pi}{4}\right]$; if $\theta_h \in \left[-\frac{\pi}{2},-\frac{\pi}{4}\right]$ then $\theta_{x_B^{br}} \in \left[-\frac{3\pi}{4},\theta_h\right]$; if $\theta_h \in \left[\frac{\pi}{4},\frac{\pi}{2}\right]$ then $\theta_{x_B^{br}} \in \left[\frac{\pi}{4},\theta_h\right]$.*

Proof. Write the difference between platforms as $x_{B1}^{br} - x_{B2}^{br} = E\left(z_1|B\right) - E\left(z_2|B\right) = \int_Z \left(z_1-z_2\right)\frac{\Pr(B|z)}{\Pr(B)} f\left(z\right) dz$. Noting that $\Pr\left(B\right) = \frac{1}{2}$ and expressing all eight octants in terms of the first octant Z_1, this reduces to the following,

$$\begin{aligned}
& 2\int_{Z_1} [(z_1 - z_2)\Pr(B|z_1,z_2) f(z_1,z_2) + (-z_1 - z_2)\Pr(B|-z_1,z_2) f(-z_1,z_2) \\
& + (z_1 + z_2)\Pr(B|z_1,-z_2) f(z_1,-z_2) + (-z_1 + z_2)\Pr(B|-z_1,-z_2) f(-z_1,-z_2) \\
& + (z_2 - z_1)\Pr(B|z_2,z_1) f(z_2,z_1) + (-z_2 - z_1)\Pr(B|-z_2,z_1) f(-z_2,z_1) \\
& + (z_2 + z_1)\Pr(B|z_2,-z_1) f(z_2,-z_1) + (-z_2 + z_1)\Pr(B|-z_2,-z_1) f(-z_2,-z_1)]dz \\
= \; & 4\int_{Z_1} \{(z_1 - z_2)[\Pr(B|z_1,z_2) - \Pr(B|z_2,z_1)] f(z_1,z_2) \\
& + (z_1 + z_2)[\Pr(B|z_1,-z_2) - \Pr(B|-z_2,z_1)] f(z_1,z_2;-\rho)\}dz
\end{aligned}$$

where equality follows from Condition 2 and Lemma 2. Invoking Lemma 2 a second time, the two differences in brackets have the same signs as $h\cdot(z_1,z_2) - h\cdot(z_2,z_1) = (h_1 - h_2)(z_1 - z_2)$ and $h\cdot(z_1,-z_2) - h\cdot(-z_2,z_1) = (h_1 - h_2)(z_1 + z_2)$, respectively, which means that the entire expression has the same sign as $h_1 - h_2$ (since $z_1 - z_2$ and $z_1 + z_2$ are both positive in Z_1). In other words, $x_B^{br}\cdot\binom{1}{-1}$ and $x_B^{br}\cdot\binom{1}{1}$ have the same signs as $h\cdot\binom{1}{-1}$ and $h\cdot\binom{1}{1}$, respectively, so θ_h and $\theta_{x_B^{br}}$ both belong to the same quadrant: either $\left[-\frac{3\pi}{4},-\frac{\pi}{4}\right]$, $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$, $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$, or $\left[\frac{3\pi}{4},\frac{5\pi}{4}\right]$. However, the proof of Proposition 2 shows that x_B^{br} differs from h in the direction of the major diagonal. That is, $\theta_{x_B^{br}} < \theta_h$ if and only if $\frac{\pi}{4} < |\theta_h| < \frac{3\pi}{4}$. ■

Proof of Proposition 7. As in the proof of Proposition 6, best response platforms are given by $x_j^{br} = E(z|j)$ and v_h produces expected vote shares that increase in the direction of h, symmetric best-response platforms $x_A^{br} = -x_B^{br}$, and a half-space best-response strategy $v_{h^{br}}$. Also, the benefit of voting B instead of voting A has the same sign as $E(z|P,s)\cdot\tilde{x}_B$, where $\tilde{x}_B = \begin{pmatrix}(1+\lambda)x_{B1}\\(1-\lambda)x_{B2}\end{pmatrix}$.

The proof of Proposition 2 shows that $\theta_{x_B^{br}} \in \left(0,\frac{\pi}{4}\right)$, which implies that $x_{B1}^{br} > x_{B2}^{br} > 0$, and therefore that $(1+\lambda)x_{B1}^{br} > (1-\lambda)x_{B2}^{br} > 0$, or $\tilde{x}_{B1}^{br} > \tilde{x}_{B2}^{br} > 0$. Thus, $\theta_{\tilde{x}_B^{br}} \in \left(0,\frac{\pi}{4}\right)$. The proof of Proposition 1 shows that if $\theta_{x_B} = 0$ and $\theta_s = \frac{\pi}{2}$ then $E(z|P,s)\cdot x_B = 0$. For the same signal realization, then, $\theta_{\tilde{x}_B^{br}} > 0$ implies that

$E(z|P,s)\cdot\tilde{x}_B^{br}>0$. In other words, a voter whose signal is orthogonal to h prefers voting B, and a voter who is indifferent between voting A and B has a signal with polar angle $\theta_s>\frac{\pi}{2}$. Thus, the half-space voting strategy that best responds to $\tilde{x}_B^{br}(v_h)$ has a normal vector h^{br} with $\theta_{h^{br}}>0$.

If $\theta_h=\frac{\pi}{4}$ then the proof of Proposition 2 shows that $\theta_{x_B^{br}}=\frac{\pi}{4}$, which implies that $x_{B1}^{br}=x_{B2}^{br}>0$, and therefore that $(1+\lambda)\,x_{B1}^{br}>(1-\lambda)\,x_{B2}^{br}>0$, or $\tilde{x}_{B1}^{br}>\tilde{x}_{B2}^{br}>0$. Thus, again $\theta_{\tilde{x}_B^{br}}\in\left(0,\frac{\pi}{4}\right)$. The proof of Proposition 1 shows that if $\theta_{x_B}=\frac{\pi}{4}$ and $\theta_s=\frac{3\pi}{4}$ then $E(z|P,s)\cdot x_B=0$. For the same signal realization, then, $\theta_{\tilde{x}_B^{br}}<\frac{\pi}{4}$ implies that $E(z|P,s)\cdot\tilde{x}_B^{br}<0$. In other words, a voter whose signal is orthogonal to h prefers voting A, and a voter who is indifferent between voting A and B has a signal with $\theta_s<\frac{3\pi}{4}$. Thus, the half-space voting strategy that best responds to $\tilde{x}_B^{br}$ has a normal vector h^{br} with $\theta_{h^{br}}<\frac{\pi}{4}$. Since $\theta_{h^{br}(h)}$ is a continuous function of θ_h, the results that $\theta_{h^{br}(h)}>\theta_h$ for $\theta_h=0$ and $\theta_{h^{br}(h)}<\theta_h$ for $\theta_h=\frac{\pi}{4}$ together imply (by the intermediate value theorem) the existence of $\theta_{h^+}\in\left(0,\frac{\pi}{4}\right)$ such that $\theta_{h^{br}(h^+)}=\theta_{h^+}$, implying that v_{h^+} is a best response to $\left(v_{h^+},x_A^+,x_B^+\right)$, and the latter constitutes a half-space equilibrium.

If $\theta_h=-\frac{\pi}{4}$ then the proof of Proposition 2 shows that $\theta_{x_B^{br}}=-\frac{\pi}{4}$, which implies that $x_{B1}^{br}>0>x_{B2}^{br}$ and $\left|x_{B1}^{br}\right|=\left|x_{B2}^{br}\right|$, and therefore that $(1+\lambda)\,x_{B1}^{br}>0>(1-\lambda)\,x_{B2}^{br}$ and $\left|(1+\lambda)\,x_{B1}^{br}\right|=\left|(1-\lambda)\,x_{B2}^{br}\right|$, or $\tilde{x}_{B1}^{br}>0>\tilde{x}_{B2}^{br}$ with $\left|\tilde{x}_{B1}^{br}\right|>\left|\tilde{x}_{B2}^{br}\right|$. Thus, $\theta_{\tilde{x}_B^{br}}\in\left(-\frac{\pi}{4},0\right)$. For the same voting strategy, the proof of Proposition 1 shows that if $\theta_{x_B}=-\frac{\pi}{4}$ and a citizen's signal realization has polar angle $\theta_s=\frac{\pi}{4}$ orthogonal to h then $E(z|P,s)\cdot x_B=0$. For the same signal realization, then, $\theta_{\tilde{x}_{br}}>-\frac{\pi}{4}$ implies that $E(z|P,s)\cdot\tilde{x}_B^{br}>0$. In other words, a voter whose signal is orthogonal to h prefers voting B, and one who is indifferent between voting A and B has a signal with $\theta_s>\frac{\pi}{4}$. Thus, the half-space voting strategy that best responds to $\tilde{x}_B^{br}$ has a normal vector h^{br} with $\theta_{h^{br}}>-\frac{\pi}{4}$.

If $\theta_h=-\frac{\pi}{2}$ then the proof of Proposition 2 shows that $\theta_{x_B^{br}}\in\left(-\frac{3\pi}{4},-\frac{\pi}{2}\right)$, which implies that $x_{B1}^{br}<0$ and $x_{B2}^{br}<0$, and therefore that $(1+\lambda)\,x_{B1}^{br}<0$ and $(1-\lambda)\,x_{B2}^{br}<$

0, or $\tilde{x}_{B1}^{br} < 0$ and $\tilde{x}_{B2}^{br} < 0$. Thus, $\theta_{\tilde{x}_B^{br}} < -\frac{\pi}{2}$. For the same voting strategy, the proof of Proposition 1 shows that if $\theta_{x_B} = -\frac{\pi}{4}$ and a citizen's signal realization has polar angle $\theta_s = 0$ orthogonal to h then $E(z|P,s) \cdot x = 0$. For the same signal realization, then, $\theta_{\tilde{x}_{br}} < -\frac{\pi}{4}$ implies that $E(z|P,s) \cdot \tilde{x}_B^{br} < 0$. In other words, a voter whose signal is orthogonal to h prefers voting A, and one who is indifferent between voting A and B has a signal with $\theta_s < 0$. Thus, the half-space voting strategy that best responds to $\tilde{x}_B^{br}(v_h)$ has a normal vector h^{br} with $\theta_{h^{br}} < -\frac{\pi}{2}$. Since $\theta_{h^{br}(h)}$ is a continuous function of θ_h, the results that $\theta_{h^{br}(h)} > \theta_h$ for $\theta_h = 0$ and $\theta_{h^{br}(h)} < \theta_h$ for $\theta_h = \frac{\pi}{4}$ together imply (by the intermediate value theorem) the existence of $\theta_{h^-} \in \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right)$ such that $\theta_{h^{br}(h^-)} = \theta_{h^-}$, implying that v_{h^-} is a best response to $\left(v_{h^-}, x_A^-, x_B^-\right)$, and the latter constitutes a half-space equilibrium. ■

Proof of Proposition 5. For any n, Parts 1 and 3 of Lemma 1 together guarantee that $\phi(j|z; v_{h^+}) > \frac{1}{2}$ for any $z \in Z_j^+$ so, by the standard jury theorem, $\lim_{n\to\infty} \Pr(j|z; v_{h^+}) = 1$ in that case. Integrating over Z_j^+ (and applying analogous logic to the case of Z_j^-) then yields the first result. The second follows from Bayes' rule, as $f(z|w=j; E_n^+) = \frac{\Pr\left(w=j|z;E_n^+\right)f(z)}{\Pr\left(w=j;E_n^+\right)} = 2\Pr(w=j|z;E_n^+)\, f(z)$, so $\lim_{n\to\infty} f(z|w=j;E_n^+) = 2f(z)\,\mathbf{1}_{z\in Z_j^+}$, and $\lim_{n\to\infty} x_{j,n}^- = \lim_{n\to\infty} E(z|w=j;E_n^+) = E\left(z|Z_j^+\right)$. Similar logic applies for Z_j^-. ■

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